Visualize Fractions Beginning Level
By the end of this section, you will be able to:

- Understand the meaning of fractions
- Model improper fractions and mixed numbers

Note:
Before you get started, take this readiness quiz.

1. Divide $1{,}439 \div 4$. If you missed this problem, review [link].

Understand the Meaning of Fractions

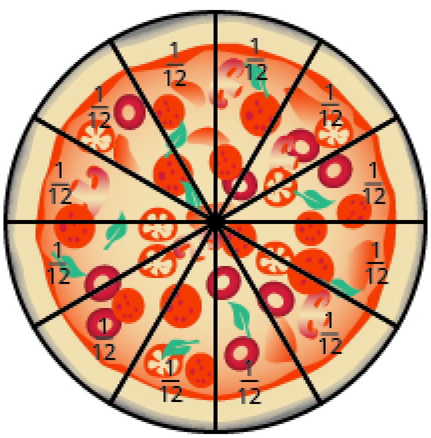
Andy and Bobby love pizza. On Monday night, they share a pizza equally. How much of the pizza does each one get? Are you thinking that each boy gets half of the pizza? That's right. There is one whole pizza, evenly divided into two parts, so each boy gets one of the two equal parts.

In math, we write $\frac{1}{2}$ to mean one out of two parts.

On Tuesday, Andy and Bobby share a pizza with their parents, Fred and Christy, with each person getting an equal amount of the whole pizza. How much of the pizza does each person get? There is one whole pizza, divided

evenly into four equal parts. Each person has one of the four equal parts, so each has $\frac{1}{4}$ of the pizza.

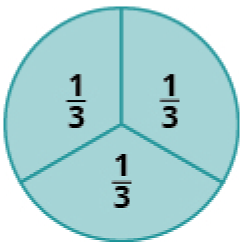
On Wednesday, the family invites some friends over for a pizza dinner. There are a total of 12 people. If they share the pizza equally, each person would get $\frac{1}{12}$ of the pizza.



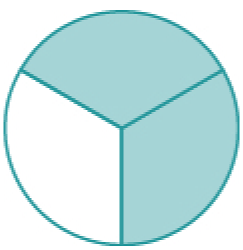
Note:
Fractions
A fraction is written $\frac{a}{b}$, where a and b are integers and $b \neq 0$. In a fraction, a is called the numerator and b is called the denominator.

A fraction is a way to represent parts of a whole. The denominator b represents the number of equal parts the whole has been divided into, and the numerator a represents how many parts are included. The denominator, b, cannot equal zero because division by zero is undefined.

In [link], the circle has been divided into three parts of equal size. Each part represents $\frac{1}{3}$ of the circle. This type of model is called a fraction circle. Other shapes, such as rectangles, can also be used to model fractions.



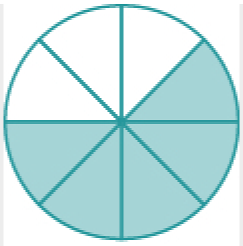
What does the fraction $\frac{2}{3}$ represent? The fraction $\frac{2}{3}$ means two of three equal parts.



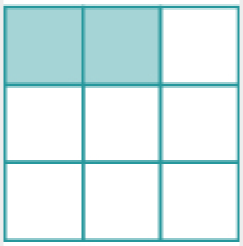
Example:
Exercise:

Problem:

Name the fraction of the shape that is shaded in each of the figures.



(a)



(b)

Solution:
Solution

We need to ask two questions. First, how many equal parts are there? This will be the denominator. Second, of these equal parts, how many are shaded? This will be the numerator.

ⓐ

How many equal parts are there?	There are eight equal parts.
How many are shaded?	Five parts are shaded.

Five out of eight parts are shaded. Therefore, the fraction of the circle that is shaded is $\frac{5}{8}$.

ⓑ

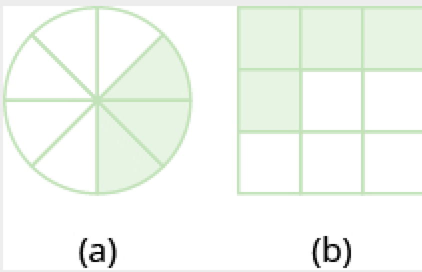
How many equal parts are there?	There are nine equal parts.
How many are shaded?	Two parts are shaded.

Two out of nine parts are shaded. Therefore, the fraction of the square that is shaded is $\frac{2}{9}$.

Note:
Exercise:

Problem:

Name the fraction of the shape that is shaded in each figure:



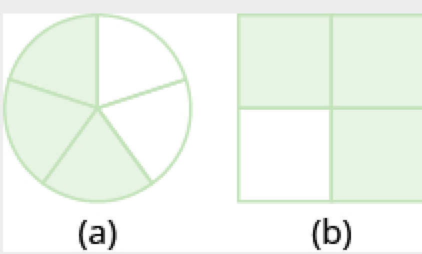
Solution:

ⓐ $\frac{3}{8}$
ⓑ $\frac{4}{9}$

Note:
Exercise:

Problem:

Name the fraction of the shape that is shaded in each figure:



Solution:

ⓐ $\frac{3}{5}$
ⓑ $\frac{3}{4}$

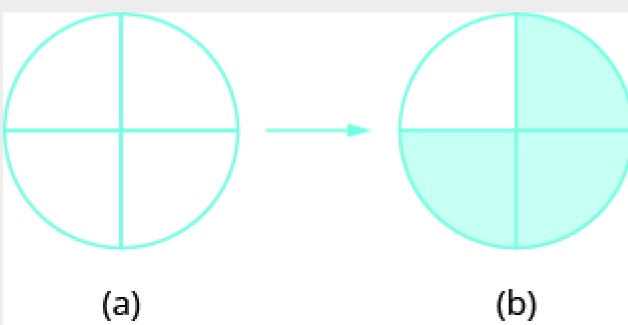
Example:
Exercise:

Problem: Shade $\frac{3}{4}$ of the circle.

Solution:
Solution

The denominator is 4, so we divide the circle into four equal parts ⓐ.

The numerator is 3, so we shade three of the four parts ⓑ.

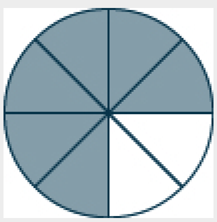


$\frac{3}{4}$ of the circle is shaded.

Note:
Exercise:

Problem: Shade $\frac{6}{8}$ of the circle.

Solution:

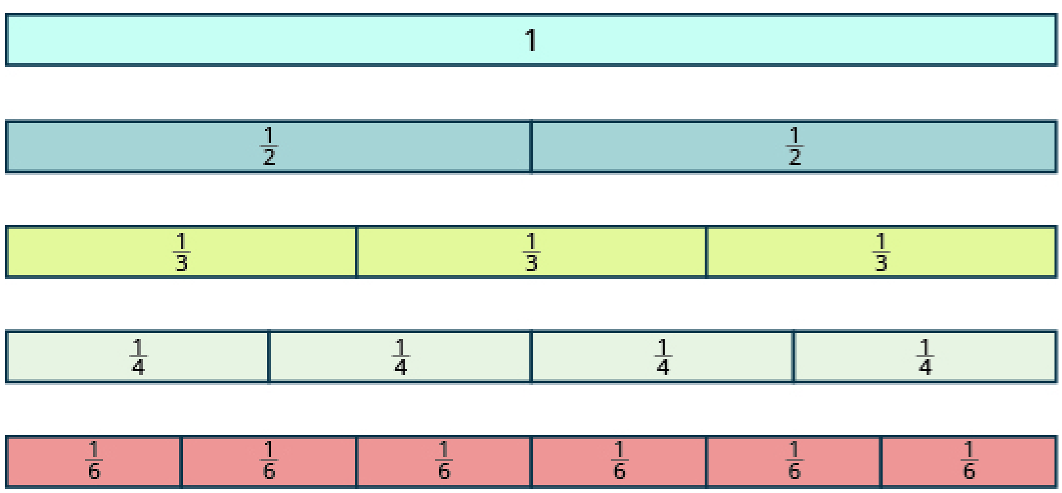


Note:
Exercise:

Problem: Shade $\frac{2}{5}$ of the rectangle.

Solution:

In [link] and [link], we used circles and rectangles to model fractions. Fractions can also be modeled as manipulatives called fraction towers, as shown in [link]. Here, the whole is modeled as one long, undivided rectangular tower. Beneath it are towers of equal length divided into different numbers of equally sized parts.



We’ll be using fraction towers to discover some basic facts about fractions. Refer to [link] to answer the following questions:

How many $\frac{1}{2}$ towers does it take to make one whole tower?	It takes two halves to make a whole, so two out of two is $\frac{2}{2}=1$.
How many $\frac{1}{3}$ towers does it take to make one whole tower?	It takes three thirds, so three out of three is $\frac{3}{3}=1$.
How many $\frac{1}{4}$ towers does it take to make one whole tower?	It takes four fourths, so four out of four is $\frac{4}{4}=1$.
How many $\frac{1}{6}$ towers does it take to make one whole tower?	It takes six sixths, so six out of six is $\frac{6}{6}=1$.
What if the whole were divided into 24 equal parts? (We have not shown fraction towers to represent this, but try to visualize it in your mind.) How many $\frac{1}{24}$ towers does it take to make one whole tower?	It takes 24 twenty-fourths, so $\frac{24}{24}=1$.

It takes 24 twenty-fourths, so $\frac{24}{24}=1$.

This leads us to the *Property of One*.

Note:
Property of One
Any number, except zero, divided by itself is one.
Equation:

$$\frac{a}{a} = 1 \qquad\qquad (a \neq 0)$$

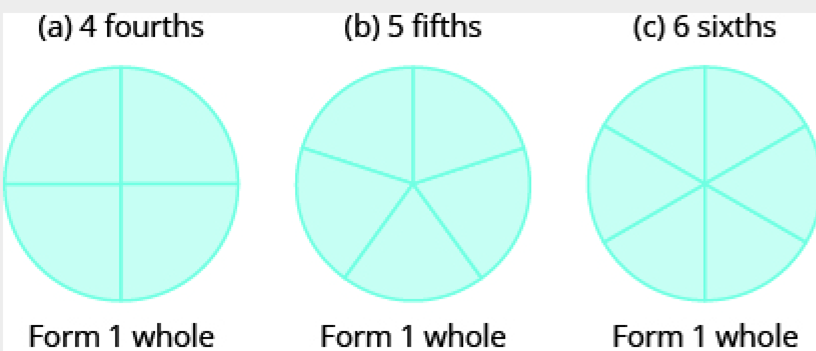
Example:
Exercise:

Problem:

Use fraction circles to make wholes using the following pieces:

ⓐ 4 fourths
ⓑ 5 fifths
ⓒ 6 sixths

Solution:
Solution



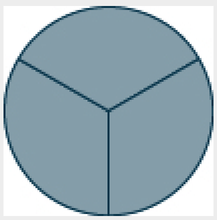
Note:

Exercise:

Problem:

Use fraction circles to make wholes with the following pieces: 3 thirds.

Solution:



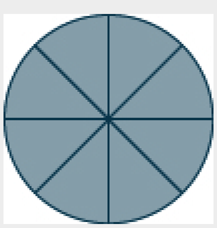
Note:

Exercise:

Problem:

Use fraction circles to make wholes with the following pieces: 8 eighths.

Solution:



What if we have more fraction pieces than we need for 1 whole? We'll look at this in the next example.

Example:
Exercise:

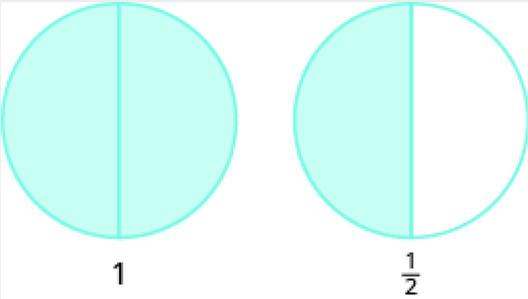
Problem:

Use fraction circles to make wholes using the following pieces:

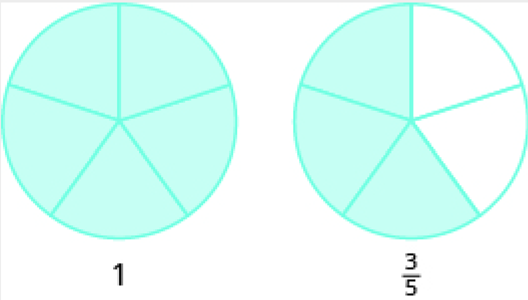
ⓐ 3 halves
ⓑ 8 fifths
ⓒ 7 thirds

Solution:
Solution

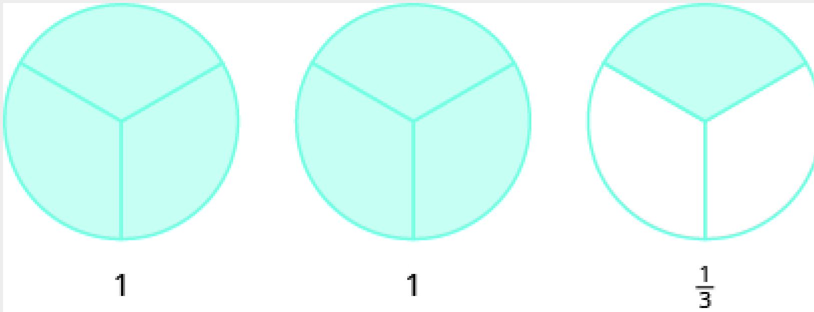
ⓐ3 halves make 1 whole with 1 half left over.



ⓑ8 fifths make 1 whole with 3 fifths left over.



ⓒ7 thirds make 2 wholes with 1 third left over.

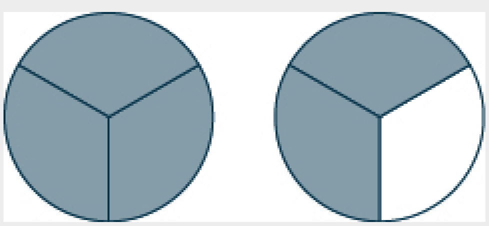


Note:
Exercise:

Problem:

Use fraction circles to make wholes with the following pieces: 5 thirds.

Solution:

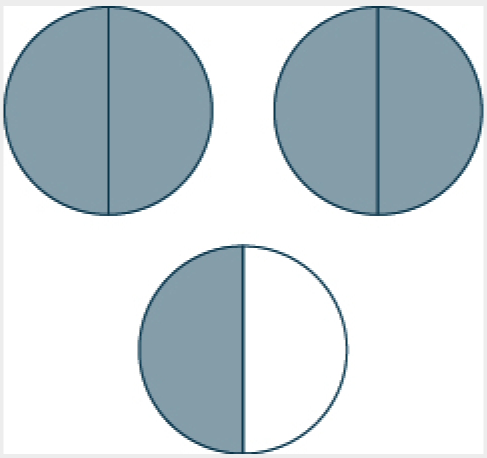


Note:
Exercise:

Problem:

Use fraction circles to make wholes with the following pieces: 5 halves.

Solution:



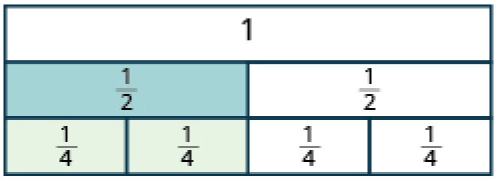
Model Equivalent Fractions

Let's think about Andy and Bobby and their favorite food again. If Andy eats $\frac{1}{2}$ of a pizza and Bobby eats $\frac{2}{4}$ of the pizza, have they eaten the same amount of pizza? In other words, does $\frac{1}{2} = \frac{2}{4}$? We can use fraction towers to find out whether Andy and Bobby have eaten *equivalent* parts of the pizza.

Note:
Equivalent Fractions
Equivalent fractions are fractions that have the same value.

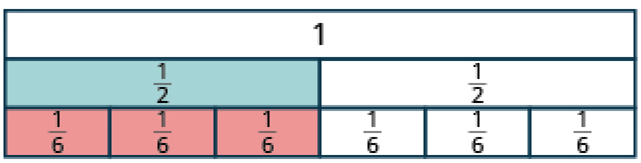
Fraction towers serve as a useful model of equivalent fractions. You may want to use fraction towers to do the following activity. Or you might make a copy of [link] and extend it to include eighths, tenths, and twelfths.

Start with a $\frac{1}{2}$ tower. How many fourths equal one-half? How many of the $\frac{1}{4}$ towers is exactly the same length as the $\frac{1}{2}$ tower?



Since two $\frac{1}{4}$ towers is the same as the $\frac{1}{2}$ tower, we see that $\frac{2}{4}$ is the same as $\frac{1}{2}$, or $\frac{2}{4} = \frac{1}{2}$.

How many of the $\frac{1}{6}$ towers is the same as the $\frac{1}{2}$ tower?



Since three $\frac{1}{6}$ towers is the same as the $\frac{1}{2}$ tower, we see that $\frac{3}{6}$ is the same as $\frac{1}{2}$.

So, $\frac{3}{6} = \frac{1}{2}$. The fractions are equivalent fractions.

Example:
Exercise:

Problem:

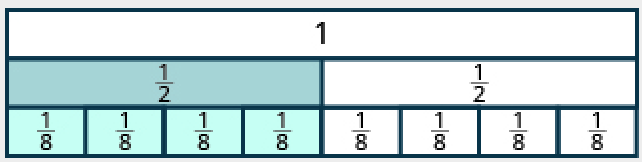
Use fraction towers to find equivalent fractions. Show your result with a figure.

ⓐ How many eighths equal one-half?
ⓑ How many tenths equal one-half?

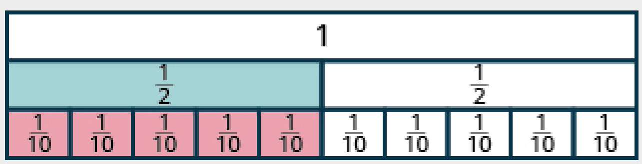
ⓒ How many twelfths equal one-half?

Solution:
Solution

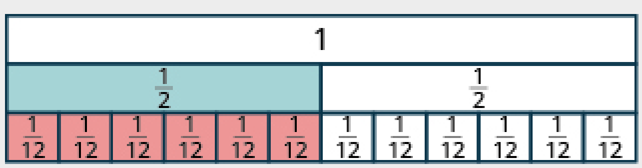
ⓐ It takes four $\frac{1}{8}$ towers to be exactly the same as the $\frac{1}{2}$ tower, so $\frac{4}{8} = \frac{1}{2}$.



ⓑ It takes five $\frac{1}{10}$ towers to be exactly the same as the $\frac{1}{2}$ tower, so $\frac{5}{10} = \frac{1}{2}$.



ⓒ It takes six $\frac{1}{12}$ towers to be exactly the same as the $\frac{1}{2}$ tower, so $\frac{6}{12} = \frac{1}{2}$.



Suppose you had towers marked $\frac{1}{20}$. How many of them would it take to equal $\frac{1}{2}$? Are you thinking ten towers? If you are, you're right, because $\frac{10}{20} = \frac{1}{2}$.

We have shown that $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}$, and $\frac{10}{20}$ are all equivalent fractions.

Note:
Exercise:

Problem:

Use fraction towers to find equivalent fractions: How many eighths equal one-fourth?

Solution:

2

Note:
Exercise:

Problem:

Use fraction towers to find equivalent fractions: How many twelfths equal one-fourth?

Solution:

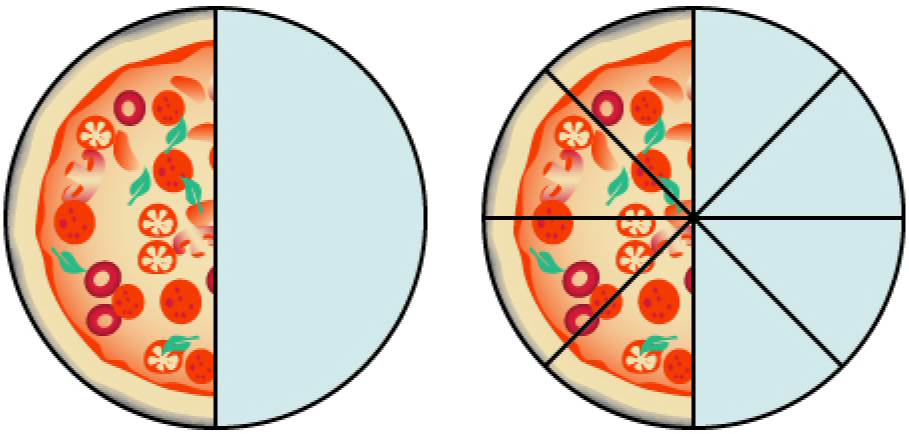
3

Find Equivalent Fractions

We used fraction towers to show that there are many fractions equivalent to $\frac{1}{2}$. For example, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{4}{8}$ are all equivalent to $\frac{1}{2}$. When we lined up the fraction towers, it took four of the $\frac{1}{8}$ towers to make the same length as a $\frac{1}{2}$ tower. This showed that $\frac{4}{8}=\frac{1}{2}$. See [link].

We can show this with pizzas, too. [link](a) shows a single pizza, cut into two equal pieces with $\frac{1}{2}$ shaded. [link](b) shows a second pizza of the same

size, cut into eight pieces with $\frac{4}{8}$ shaded.



$\frac{1}{2}$ (a) $\frac{4}{8}$ (b)

This is another way to show that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$.

How can we use mathematics to change $\frac{1}{2}$ into $\frac{4}{8}$? How could you take a pizza that is cut into two pieces and cut it into eight pieces? You could cut each of the two larger pieces into four smaller pieces! The whole pizza would then be cut into eight pieces instead of just two. Mathematically, what we've described could be written as:

$$\frac{1 \cdot 4}{2 \cdot 4} = \frac{4}{8}$$

These models lead to the Equivalent Fractions Property, which states that if we multiply the numerator and denominator of a fraction by the same number, the value of the fraction does not change.

Note:
Equivalent Fractions Property
If a, b, and c are numbers where $b \neq 0$ and $c \neq 0$, then
Equation:

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$$

When working with fractions, it is often necessary to express the same fraction in different forms. To find equivalent forms of a fraction, we can use the Equivalent Fractions Property. For example, consider the fraction one-half.

$$\frac{1 \cdot 3}{2 \cdot 3} = \frac{3}{6} \quad \text{so} \quad \frac{1}{2} = \frac{3}{6}$$

$$\frac{1 \cdot 2}{2 \cdot 2} = \frac{2}{4} \quad \text{so} \quad \frac{1}{2} = \frac{2}{4}$$

$$\frac{1 \cdot 10}{2 \cdot 10} = \frac{10}{20} \quad \text{so} \quad \frac{1}{2} = \frac{10}{20}$$

So, we say that $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{10}{20}$ are equivalent fractions.

Example:
Exercise:

Problem: Find three fractions equivalent to $\frac{2}{5}$.

Solution:
Solution

To find a fraction equivalent to $\frac{2}{5}$, we multiply the numerator and denominator by the same number (but not zero). Let us multiply them by 2, 3, and 5.

$$\frac{2 \cdot 2}{5 \cdot 2} = \frac{4}{10} \qquad \frac{2 \cdot 3}{5 \cdot 3} = \frac{6}{15} \qquad \frac{2 \cdot 5}{5 \cdot 5} = \frac{10}{25}$$

So, $\frac{4}{10}$, $\frac{6}{15}$, and $\frac{10}{25}$ are equivalent to $\frac{2}{5}$.

Note:
Exercise:

Problem: Find three fractions equivalent to $\frac{3}{5}$.

Solution:

Correct answers include $\frac{6}{10}$, $\frac{9}{15}$, and $\frac{12}{20}$.

Note:
Exercise:

Problem: Find three fractions equivalent to $\frac{4}{5}$.

Solution:

Correct answers include $\frac{8}{10}$, $\frac{12}{15}$, and $\frac{16}{20}$.

Example:
Exercise:

Problem:

Find a fraction with a denominator of 21 that is equivalent to $\frac{2}{7}$.

Solution:
Solution

To find equivalent fractions, we multiply the numerator and denominator by the same number. In this case, we need to multiply the denominator by a number that will result in 21.

Since we can multiply 7 by 3 to get 21, we can find the equivalent fraction by multiplying both the numerator and denominator by 3.

$$\frac{2}{7} = \frac{2 \cdot 3}{7 \cdot 3} = \frac{6}{21}$$

Note:
Exercise:

Problem:

Find a fraction with a denominator of 21 that is equivalent to $\frac{6}{7}$.

Solution:

$\frac{18}{21}$

Note:
Exercise:

Problem:

Find a fraction with a denominator of 100 that is equivalent to $\frac{3}{10}$.

Solution:

$\frac{30}{100}$

Simplifying Fractions

Recall the Equivalent Fractions Property:

If a, b, and c are numbers where $b \neq 0$ and $c \neq 0$, then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$

We have used this property to generate equivalent fractions where the numerator and denominator are greater than those on the original fraction. An equation always works in both directions so we can also use the property to make equivalent fractions where the numerator and denominator are less than those on the original fraction.

When we replace a fraction with an equivalent fraction that a smaller numerator and denominator than those of the original fraction, we say that we have reduced the fraction.
For example, if we replace $\frac{10}{12}$ with $\frac{5}{6}$. We can do this because $\frac{10}{12} = \frac{5 \cdot 2}{6 \cdot 2}$ and therefore the Equivalent Fractions Property applies.
Sometimes this is called reducing the fraction. Remember that while the numerator and denominator are both smaller (they have been reduced) the value of the fraction has not changed.

A fraction that has been simplified as much as possible is said to be in lowest terms.
$\frac{5}{6}$ is in lowest terms because there is no factor common to both 5 and 6. 10 and 12 had the common factor 2.

One way to find a common factor is to search for a number that will divide both the numerator and denominator without a remainder. Notice that 2 divides both 10 and 12 with no remainder.

Simplifying a fraction is related to what is commonly called canceling. For $\frac{10}{12} = \frac{5 \cdot 2}{6 \cdot 2}$ the common factor 2 is often crossed out in both the numerator and denominator. The Equivalent Fractions Property tells us that it is OK to do this.

Example:
Simplify $\frac{18}{24}$ to lowest terms.

Solution:
$\frac{18}{24} = \frac{3 \cdot 6}{4 \cdot 6}$. By the Equivalent Fractions Property that simplifies to $\frac{3}{4}$.

Exercise:

Simplify each fraction to lowest terms:

a) $\frac{15}{20}$

b) $\frac{20}{30}$

c) $\frac{7}{14}$

Problem: d) $\frac{18}{60}$

Solution:

Simplify each fraction to lowest terms:

a) $\frac{15}{20} = \frac{3 \cdot 5}{4 \cdot 5} = \frac{3}{4}$

b) $\frac{20}{30} = \frac{2 \cdot 10}{3 \cdot 10} = \frac{2}{3}$

c) $\frac{7}{14} = \frac{1 \cdot 7}{2 \cdot 7} = \frac{1}{2}$

d) $\frac{18}{60} = \frac{3 \cdot 6}{10 \cdot 6} = \frac{1}{2}$

Model Improper Fractions and Mixed Numbers

In [link] (b), you had eight equal fifth pieces. You used five of them to make one whole, and you had three fifths left over. Let us use fraction notation to show what happened. You had eight pieces, each of them one fifth, $\frac{1}{5}$, so altogether you had eight fifths, which we can write as $\frac{8}{5}$. The fraction $\frac{8}{5}$ is one whole, 1, plus three fifths, $\frac{3}{5}$, or $1\frac{3}{5}$, which is read as *one and three-fifths*.

The number $1\frac{3}{5}$ is called a mixed number. A mixed number consists of a whole number and a fraction.

Note:
Mixed Numbers
A **mixed number** consists of a whole number a and a fraction $\frac{b}{c}$ where $c \neq 0$. It is written as follows.
Equation:

$$a\frac{b}{c} \qquad c \neq 0$$

Fractions such as $\frac{5}{4}$, $\frac{3}{2}$, $\frac{5}{5}$, and $\frac{7}{3}$ are called improper fractions. In an improper fraction, the numerator is greater than or equal to the denominator, so its value is greater than or equal to one. When a fraction has a numerator that is smaller than the denominator, it is called a proper fraction, and its value is less than one. Fractions such as $\frac{1}{2}$, $\frac{3}{7}$, and $\frac{11}{18}$ are proper fractions.

Note:
Proper and Improper Fractions
The fraction $\frac{a}{b}$ is a **proper fraction** if $a < b$ and an **improper fraction** if $a \geq b$.

Example:
Exercise:

Problem:

Name the improper fraction modeled. Then write the improper fraction as a mixed number.

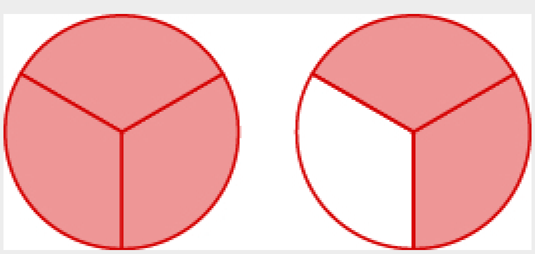
Solution:
Solution

Each circle is divided into three pieces, so each piece is $\frac{1}{3}$ of the circle. There are four pieces shaded, so there are four thirds or $\frac{4}{3}$. The figure shows that we also have one whole circle and one third, which is $1\frac{1}{3}$. So, $\frac{4}{3} = 1\frac{1}{3}$.

Note:
Exercise:

Problem:

Name the improper fraction. Then write it as a mixed number.



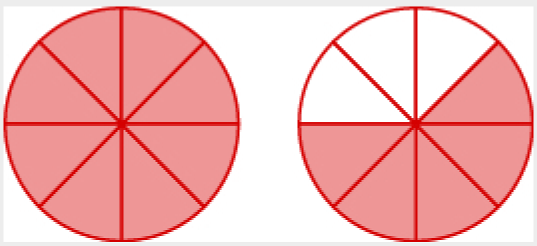
Solution:

$\frac{5}{3} = 1\frac{2}{3}$

Note:
Exercise:

Problem:

Name the improper fraction. Then write it as a mixed number.



Solution:

$\frac{13}{8} = 1\frac{5}{8}$

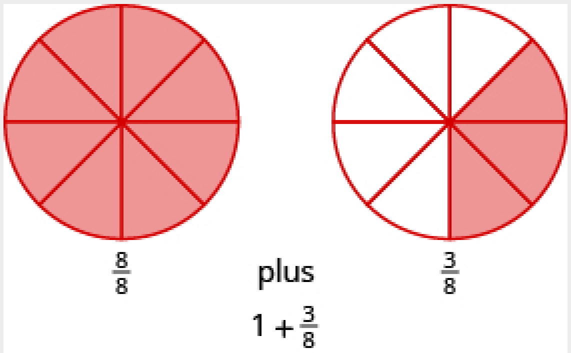
Example:
Exercise:

Problem: Draw a figure to model $\frac{11}{8}$.

Solution:
Solution

The denominator of the improper fraction is 8. Draw a circle divided into eight pieces and shade all of them. This takes care of eight

eighths, but we have 11 eighths. We must shade three of the eight parts of another circle.

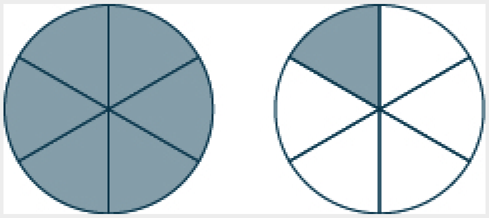


So, $\frac{11}{8} = 1\frac{3}{8}$.

Note:
Exercise:

Problem: Draw a figure to model $\frac{7}{6}$.

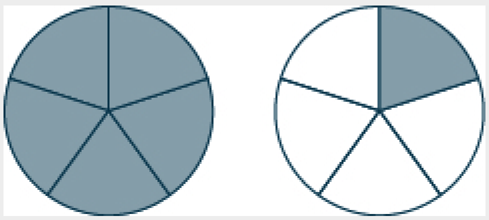
Solution:



Note:
Exercise:

Problem: Draw a figure to model $\frac{6}{5}$.

Solution:



Example:
Exercise:

Problem:

Use a model to rewrite the improper fraction $\frac{11}{6}$ as a mixed number.

Solution:
Solution

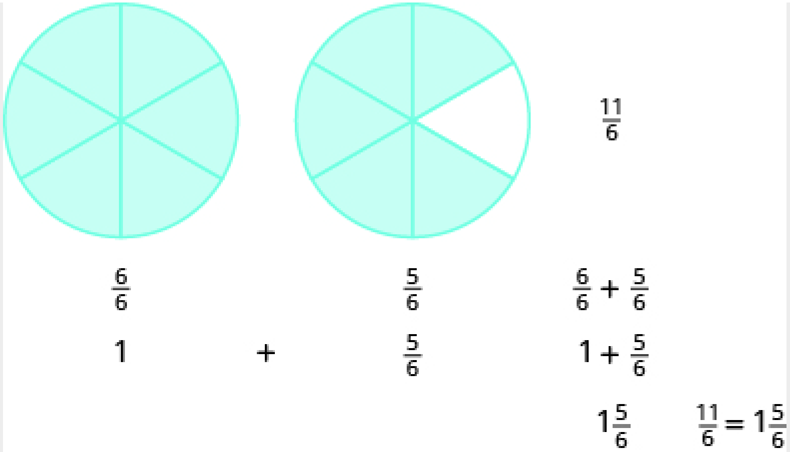
We start with 11 sixths $\left(\frac{11}{6}\right)$. We know that six sixths makes one whole.

Equation:

$$\frac{6}{6} = 1$$

That leaves us with five more sixths, which is $\frac{5}{6}$ $\left(11 \text{ sixths minus } 6 \text{ sixths is } 5 \text{ sixths}\right)$.

So, $\frac{11}{6} = 1\frac{5}{6}$.



Note:
Exercise:

Problem:

Use a model to rewrite the improper fraction as a mixed number: $\frac{9}{7}$.

Solution:

$1\frac{2}{7}$

Note:
Exercise:

Problem:

Use a model to rewrite the improper fraction as a mixed number: $\frac{7}{4}$.

Solution:

$1\frac{3}{4}$

Example:
Exercise:

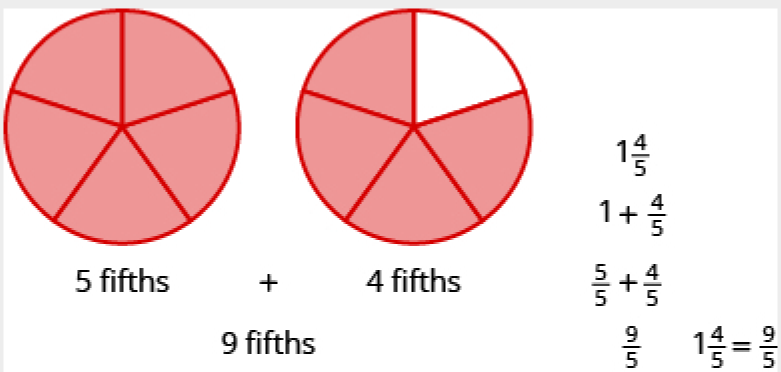
Problem:

Use a model to rewrite the mixed number $1\frac{4}{5}$ as an improper fraction.

Solution:
Solution

The mixed number $1\frac{4}{5}$ means one whole plus four fifths. The denominator is 5, so the whole is $\frac{5}{5}$. Together five fifths and four fifths equals nine fifths.

So, $1\frac{4}{5} = \frac{9}{5}$.



Note:
Exercise:

Problem:

Use a model to rewrite the mixed number as an improper fraction: $1\frac{3}{8}$.

Solution:

$\frac{11}{8}$

Note:
Exercise:

Problem:

Use a model to rewrite the mixed number as an improper fraction: $1\frac{5}{6}$.

Solution:

$\frac{11}{6}$

Convert between Improper Fractions and Mixed Numbers

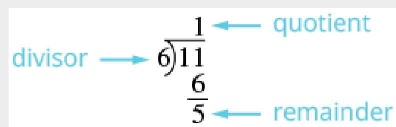
In [link], we converted the improper fraction $\frac{11}{6}$ to the mixed number $1\frac{5}{6}$ using fraction circles. We did this by grouping six sixths together to make a whole; then we looked to see how many of the 11 pieces were left. We saw that $\frac{11}{6}$ made one whole group of six sixths plus five more sixths, showing that $\frac{11}{6} = 1\frac{5}{6}$.

The division expression $\frac{11}{6}$ (which can also be written as $6\overline{)11}$) tells us to find how many groups of 6 are in 11. To convert an improper fraction to a mixed number without fraction circles, we divide.

Example:
Exercise:

Problem: Convert $\frac{11}{6}$ to a mixed number.

Solution:
Solution

	$\frac{11}{6}$
Divide the denominator into the numerator.	Remember $\frac{11}{6}$ means $11 \div 6$.
	
Identify the quotient, remainder and divisor.	
Write the mixed number as quotient $\frac{\text{remainder}}{\text{divisor}}$.	$1\frac{5}{6}$
So, $\frac{11}{6} = 1\frac{5}{6}$	

Note:
Exercise:

Problem: Convert the improper fraction to a mixed number: $\frac{13}{7}$.

Solution:

$1\frac{6}{7}$.

Note:
Exercise:

Problem: Convert the improper fraction to a mixed number: $\frac{14}{9}$.

Solution:

$1\frac{5}{9}$

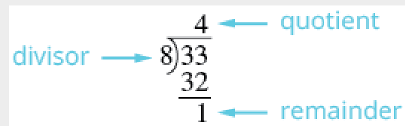
Note:
Convert an improper fraction to a mixed number.

Divide the denominator into the numerator.
Identify the quotient, remainder, and divisor.
Write the mixed number as quotient $\frac{\text{remainder}}{\text{divisor}}$.

Example:
Exercise:

Problem: Convert the improper fraction $\frac{33}{8}$ to a mixed number.

Solution:
Solution

	$\frac{33}{8}$
Divide the denominator into the numerator.	Remember, $\frac{33}{8}$ means $8\overline{)33}$.
Identify the quotient, remainder, and divisor.	
Write the mixed number as quotient $\frac{\text{remainder}}{\text{divisor}}$.	$4\frac{1}{8}$
	So, $\frac{33}{8} = 4\frac{1}{8}$

Note:
Exercise:

Problem: Convert the improper fraction to a mixed number: $\frac{23}{7}$.

Solution:

$3\frac{2}{7}$

Note:
Exercise:

Problem: Convert the improper fraction to a mixed number: $\frac{48}{11}$.

Solution:

$4\frac{4}{11}$

In [link], we changed $1\frac{4}{5}$ to an improper fraction by first seeing that the whole is a set of five fifths. So we had five fifths and four more fifths.
Equation:

$$\frac{5}{5} + \frac{4}{5} = \frac{9}{5}$$

Where did the nine come from? There are nine fifths—one whole (five fifths) plus four fifths. Let us use this idea to see how to convert a mixed number to an improper fraction.

Example:
Exercise:

Problem: Convert the mixed number $4\frac{2}{3}$ to an improper fraction.

Solution:
Solution

	$4\frac{2}{3}$
Multiply the whole number by the denominator.	

The whole number is 4 and the denominator is 3.	$\frac{4 \cdot 3 + \square}{\square}$
Simplify.	$\frac{12 + \square}{\square}$
Add the numerator to the product.	
The numerator of the mixed number is 2.	$\frac{12 + 2}{\square}$
Simplify.	$\frac{14}{\square}$
Write the final sum over the original denominator.	
The denominator is 3.	$\frac{14}{3}$

Note:
Exercise:

Problem: Convert the mixed number to an improper fraction: $3\,\frac{5}{7}$.

Solution:

$\frac{26}{7}$

Note:
Exercise:

Problem: Convert the mixed number to an improper fraction: $2\frac{7}{8}$.

Solution:

$\frac{23}{8}$

Note:
Convert a mixed number to an improper fraction.

Multiply the whole number by the denominator.
Add the numerator to the product found in Step 1.
Write the final sum over the original denominator.

Example:
Exercise:

Problem: Convert the mixed number $10\frac{2}{7}$ to an improper fraction.

Solution:
Solution

	$10\frac{2}{7}$
Multiply the whole number by the denominator.	
The whole number is 10 and the denominator is 7.	$\frac{10 \cdot 7 + \square}{\square}$
Simplify.	$\frac{70 + \square}{\square}$
Add the numerator to the product.	
The numerator of the mixed number is 2.	$\frac{70 + 2}{\square}$
Simplify.	$\frac{72}{\square}$
Write the final sum over the original denominator.	
The denominator is 7.	$\frac{72}{7}$

Note:
Exercise:

Problem: Convert the mixed number to an improper fraction: $4\frac{6}{11}$.

Solution:

$\frac{50}{11}$

Note:
Exercise:

Problem: Convert the mixed number to an improper fraction: $11\frac{1}{3}$.

Solution:

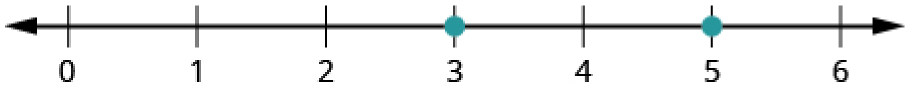
$\frac{34}{3}$

Locate Fractions and Mixed Numbers on the Number Line

Now we are ready to plot fractions on a number line. This will help us visualize fractions and understand their values.

Let us locate $\frac{1}{5}, \frac{4}{5}, 3, 3\frac{1}{3}, \frac{7}{4}, \frac{9}{2}, 5,$ and $\frac{8}{3}$ on the number line.

We will start with the whole numbers 3 and 5 because they are the easiest to plot.



The proper fractions listed are $\frac{1}{5}$ and $\frac{4}{5}$. We know proper fractions have values less than one, so $\frac{1}{5}$ and $\frac{4}{5}$ are located between the whole numbers 0

and 1. The denominators are both 5, so we need to divide the segment of the number line between 0 and 1 into five equal parts. We can do this by drawing four equally spaced marks on the number line, which we can then label as $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$.

Now plot points at $\frac{1}{5}$ and $\frac{4}{5}$.

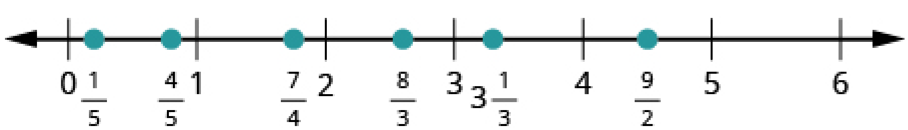
The only mixed number to plot is $3\frac{1}{3}$. Between what two whole numbers is $3\frac{1}{3}$? Remember that a mixed number is a whole number plus a proper fraction, so $3\frac{1}{3} > 3$. Since it is greater than 3, but not a whole unit greater, $3\frac{1}{3}$ is between 3 and 4. We need to divide the portion of the number line between 3 and 4 into three equal pieces (thirds) and plot $3\frac{1}{3}$ at the first mark.

Finally, look at the improper fractions $\frac{7}{4}$, $\frac{9}{2}$, and $\frac{8}{3}$. Locating these points will be easier if you change each of them to a mixed number.

Equation:

$$\frac{7}{4} = 1\frac{3}{4}, \quad \frac{9}{2} = 4\frac{1}{2}, \quad \frac{8}{3} = 2\frac{2}{3}$$

Here is the number line with all the points plotted.



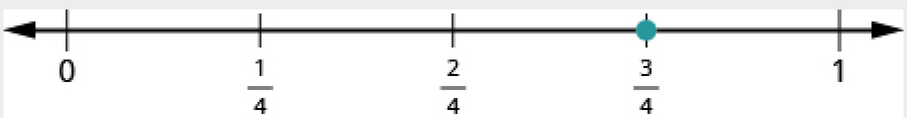
Example:
Exercise:

Problem:

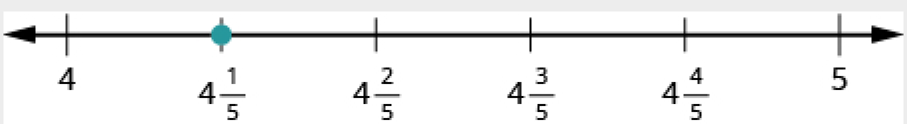
Locate and label the following on a number line: $\frac{3}{4}$, $\frac{4}{3}$, $\frac{5}{3}$, $4\frac{1}{5}$, and $\frac{7}{2}$.

Solution:
Solution

Start by locating the proper fraction $\frac{3}{4}$. It is between 0 and 1. To do this, divide the distance between 0 and 1 into four equal parts. Then plot $\frac{3}{4}$.



Next, locate the mixed number $4\frac{1}{5}$. It is between 4 and 5 on the number line. Divide the number line between 4 and 5 into five equal parts, and then plot $4\frac{1}{5}$ one-fifth of the way between 4 and 5.



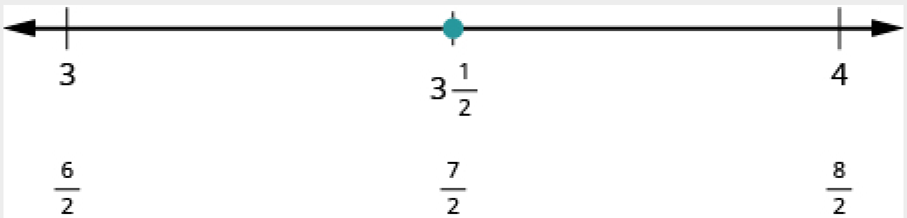
Now locate the improper fractions $\frac{4}{3}$ and $\frac{5}{3}$.

It is easier to plot them if we convert them to mixed numbers first.
Equation:

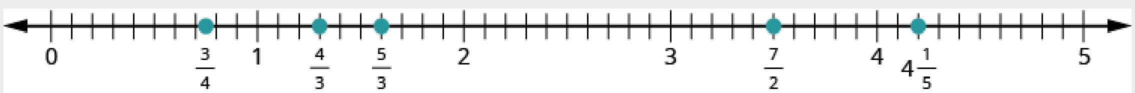
$$\frac{4}{3} = 1\frac{1}{3}, \quad \frac{5}{3} = 1\frac{2}{3}$$

Divide the distance between 1 and 2 into thirds.

Next let us plot $\frac{7}{2}$. We write it as a mixed number, $\frac{7}{2} = 3\frac{1}{2}$. Plot it between 3 and 4.



The number line shows all the numbers located on the number line.

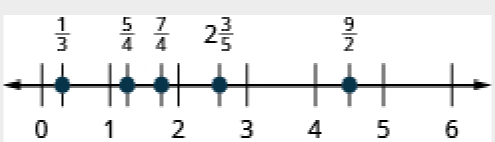


Note:
Exercise:

Problem:

Locate and label the following on a number line: $\frac{1}{3}$, $\frac{5}{4}$, $\frac{7}{4}$, $2\frac{3}{5}$, $\frac{9}{2}$.

Solution:

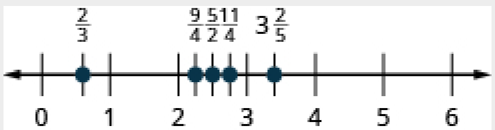


Note:
Exercise:

Problem:

Locate and label the following on a number line: $\frac{2}{3}, \frac{5}{2}, \frac{9}{4}, \frac{11}{4}, 3\frac{2}{5}$.

Solution:



Key Concepts

- **Property of One**
  - Any number, except zero, divided by itself is one.
    $\frac{a}{a} = 1$, where $a \neq 0$.
- **Equivalent Fractions Property**
  - If a, b, and c are numbers where $b \neq 0$, $c \neq 0$, then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$.
- **Mixed Numbers**
  - A **mixed number** consists of a whole number a and a fraction $\frac{b}{c}$ where $c \neq 0$.
  - It is written as follows: $a\frac{b}{c} \qquad c \neq 0$
- **Proper and Improper Fractions**

- The fraction ab is a proper fraction if $a < b$ and an improper fraction if $a \geq b$.

- **Convert an improper fraction to a mixed number.**

  Divide the denominator into the numerator.
  Identify the quotient, remainder, and divisor.
  Write the mixed number as quotient $\frac{\text{remainder}}{\text{divisor}}$.

- **Convert a mixed number to an improper fraction.**

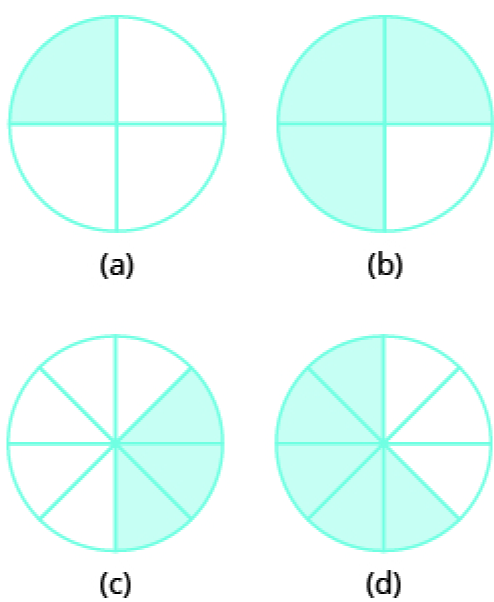
  Multiply the whole number by the denominator.
  Add the numerator to the product found in Step 1.
  Write the final sum over the original denominator.

Exercises

Practice Makes Perfect

In the following exercises, name the fraction of each figure that is shaded.
Exercise:

Problem:

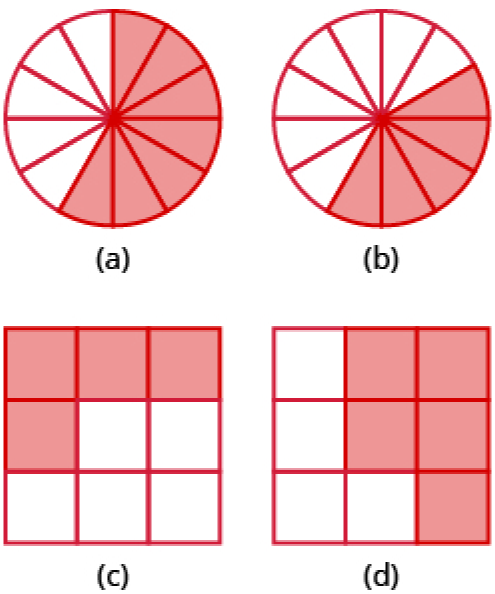


Solution:

ⓐ $\frac{1}{4}$
ⓑ $\frac{3}{4}$
ⓒ $\frac{3}{8}$
ⓓ $\frac{5}{9}$

Exercise:

Problem:

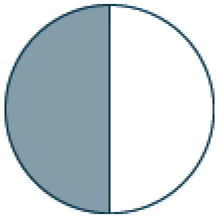


In the following exercises, shade parts of circles or squares to model the following fractions.

Exercise:

Problem: $\frac{1}{2}$

Solution:



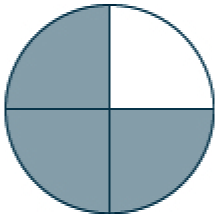
Exercise:

Problem: $\frac{1}{3}$

Exercise:

Problem: $\frac{3}{4}$

Solution:



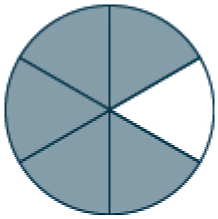
Exercise:

Problem: $\frac{2}{5}$

Exercise:

Problem: $\frac{5}{6}$

Solution:



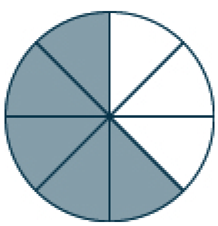
Exercise:

Problem: $\frac{7}{8}$

Exercise:

Problem: $\frac{5}{8}$

Solution:



Exercise:

Problem: $\frac{7}{10}$

In the following exercises, use fraction towers or draw a figure to find equivalent fractions.

Exercise:

Problem: How many sixths equal one-third?

Exercise:

Problem: How many twelfths equal one-third?

Solution:

4

Exercise:

Problem: How many eighths equal three-fourths?

Exercise:

Problem: How many twelfths equal three-fourths?

Solution:

9

Exercise:

Problem: How many fourths equal three-halves?

Exercise:

Problem: How many sixths equal three-halves?

Solution:

9

In the following exercises, find three fractions equivalent to the given fraction. Show your work, using figures or algebra.

Exercise:

Problem: $\frac{1}{4}$

Exercise:

Problem: $\frac{1}{3}$

Solution:

Answers may vary. Correct answers include $\frac{2}{6}, \frac{3}{9}, \frac{4}{12}$.

Exercise:

Problem: $\frac{3}{8}$

Exercise:

Problem: $\frac{5}{6}$

Solution:

Answers may vary. Correct answers include $\frac{10}{12}, \frac{15}{18}, \frac{20}{24}$.

Exercise:

Problem: $\frac{2}{7}$

Exercise:

Problem: $\frac{5}{9}$

Solution:

Answers may vary. Correct answers include $\frac{10}{18}, \frac{15}{27}, \frac{20}{36}$.

In the following exercises, simplify as much as possible.

Exercise:

Problem: $\frac{9}{12}$

Solution:

$\frac{3}{4}$

Exercise:

Problem: $\frac{8}{20}$

Solution:

$\frac{2}{5}$

Exercise:

Problem: $\frac{11}{22}$

Solution:

$\frac{1}{2}$

Exercise:

Problem: $\frac{28}{63}$

Solution:

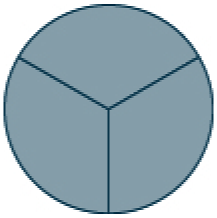
$\frac{4}{9}$

In the following exercises, use fraction circles to make wholes, if possible, with the following pieces.

Exercise:

Problem: 3 thirds

Solution:



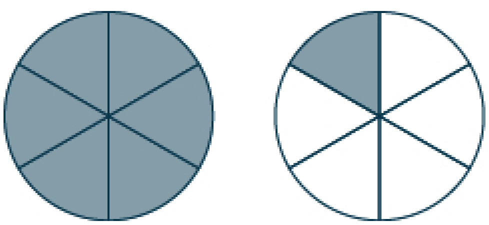
Exercise:

Problem: 8 eighths

Exercise:

Problem: 7 sixths

Solution:



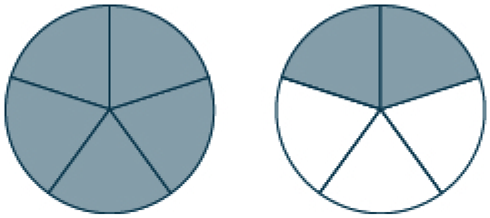
Exercise:

Problem: 4 thirds

Exercise:

Problem: 7 fifths

Solution:



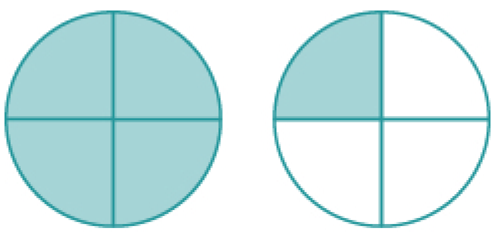
Exercise:

Problem: 7 fourths

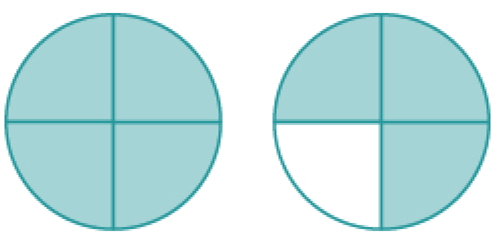
In the following exercises, name the improper fractions. Then write each improper fraction as a mixed number.

Exercise:

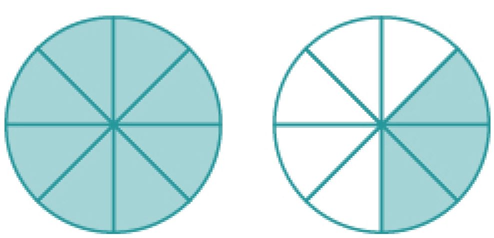
Problem:



(a)



(b)



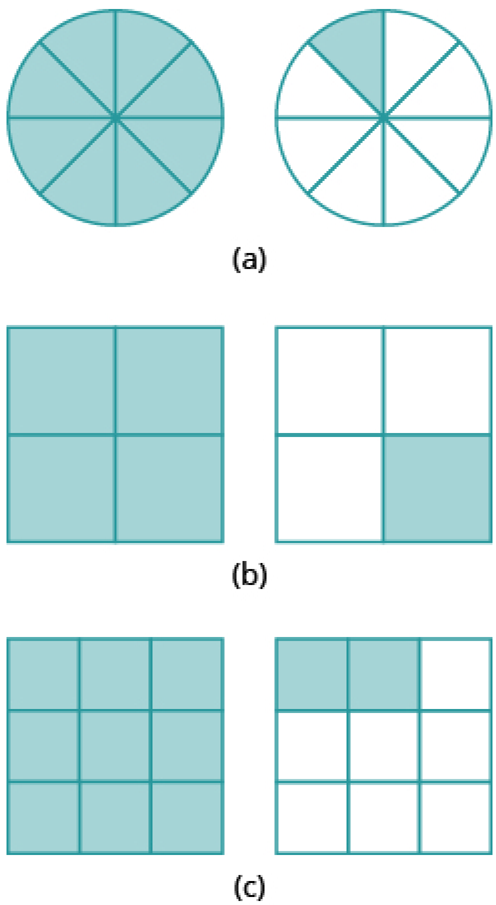
(c)

Solution:

ⓐ $\frac{5}{4} = 1\frac{1}{4}$
ⓑ $\frac{7}{4} = 1\frac{3}{4}$
ⓒ $\frac{11}{8} = 1\frac{3}{8}$

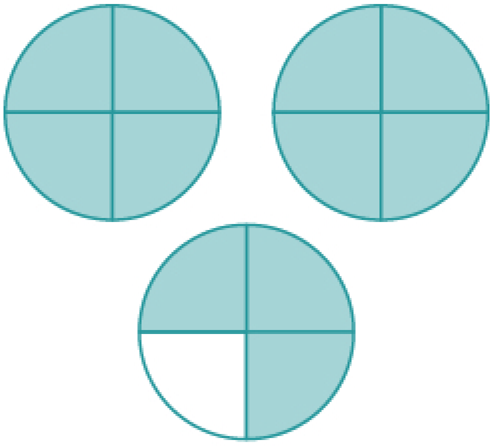
Exercise:

Problem:

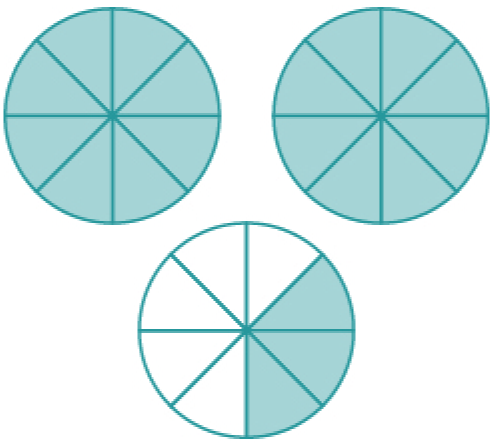


Exercise:

Problem:



(a)



(b)

Solution:

ⓐ $\frac{11}{4} = 2\frac{3}{4}$
ⓑ $\frac{19}{8} = 2\frac{3}{8}$

In the following exercises, draw fraction circles to model the given fraction.

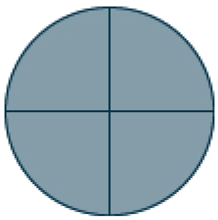
Exercise:

Problem: $\frac{3}{3}$

Exercise:

Problem: $\frac{4}{4}$

Solution:



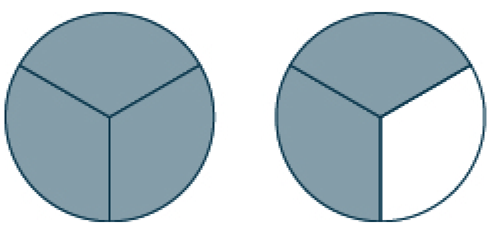
Exercise:

Problem: $\frac{7}{4}$

Exercise:

Problem: $\frac{5}{3}$

Solution:



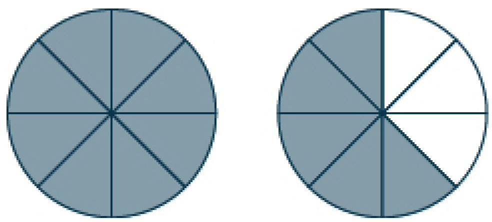
Exercise:

Problem: $\frac{11}{6}$

Exercise:

Problem: $\frac{13}{8}$

Solution:



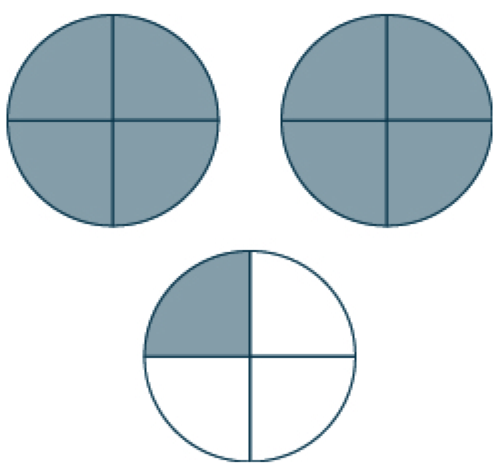
Exercise:

Problem: $\frac{10}{3}$

Exercise:

Problem: $\frac{9}{4}$

Solution:



In the following exercises, rewrite the improper fraction as a mixed number.

Exercise:

Problem: $\frac{3}{2}$

Exercise:

Problem: $\frac{5}{3}$

Solution:

$1\frac{2}{3}$

Exercise:

Problem: $\frac{11}{4}$

Exercise:

Problem: $\frac{13}{5}$

Solution:

$2\frac{3}{5}$

Exercise:

Problem: $\frac{25}{6}$

Exercise:

Problem: $\frac{28}{9}$

Solution:

$3\frac{1}{9}$

Exercise:

Problem: $\frac{42}{13}$

Exercise:

Problem: $\frac{47}{15}$

Solution:

$3\,\frac{2}{15}$

In the following exercises, rewrite the mixed number as an improper fraction.

Exercise:

Problem: $1\,\frac{2}{3}$

Exercise:

Problem: $1\,\frac{2}{5}$

Solution:

$\frac{7}{5}$

Exercise:

Problem: $2\,\frac{1}{4}$

Exercise:

Problem: $2\,\frac{5}{6}$

Solution:

$\frac{17}{6}$

Exercise:

Problem: $2\frac{7}{9}$

Exercise:

Problem: $2\frac{5}{7}$

Solution:

$\frac{19}{7}$

Exercise:

Problem: $3\frac{4}{7}$

Exercise:

Problem: $3\frac{5}{9}$

Solution:

$\frac{32}{9}$

In the following exercises, plot the numbers on a number line.

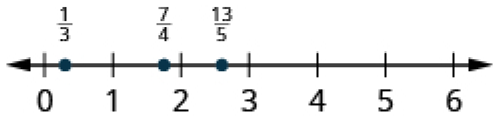
Exercise:

Problem: $\frac{2}{3}, \frac{5}{4}, \frac{12}{5}$

Exercise:

Problem: $\frac{1}{3}, \frac{7}{4}, \frac{13}{5}$

Solution:



Exercise:

Problem: $\frac{1}{4}$, $\frac{9}{5}$, $\frac{11}{3}$

Exercise:

Problem: $\frac{7}{10}$, $\frac{5}{2}$, $\frac{13}{8}$, 3

Solution:

Everyday Math

Exercise:

Problem:

Music Measures A choreographed dance is broken into counts. A $\frac{1}{1}$ count has one step in a count, a $\frac{1}{2}$ count has two steps in a count and a $\frac{1}{3}$ count has three steps in a count. How many steps would be in a $\frac{1}{5}$ count? What type of count has four steps in it?

Writing Exercises

Exercise:

Problem:

Give an example from your life experience (outside of school) where it was important to understand fractions.

Solution:

Answers will vary.

Exercise:

Problem:

Explain how you locate the improper fraction $\frac{21}{4}$ on a number line on which only the whole numbers from 0 through 10 are marked.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can…	Confidently	With some help	No-I don't get it!
understand the meaning of fractions.			
model improper fractions and mixed numbers.			
convert between improper fractions and mixed numbers.			
model equivalent fractions.			
find equivalent fractions.			
locate fractions and mixed numbers on the number line.			
order fractions and mixed numbers.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Glossary

equivalent fractions
: Equivalent fractions are two or more fractions that have the same value.

fraction

A fraction is written $\frac{a}{b}$. in a fraction, a is the numerator and b is the denominator. A fraction represents parts of a whole. The denominator b is the number of equal parts the whole has been divided into, and the numerator a indicates how many parts are included.

mixed number

A mixed number consists of a whole number a and a fraction $\frac{b}{c}$ where $c \neq 0$. It is written as $a\frac{b}{c}$, where $c \neq 0$.

proper and improper fractions

The fraction $\frac{a}{b}$ is *proper* if $a < b$and *improper* if $a > b$.

Add and Subtract Fractions with Common Denominators Beginning Level
By the end of this section, you will be able to:

- Model fraction addition
- Add fractions with a common denominator
- Model fraction subtraction
- Subtract fractions with a common denominator

Note:
Before you get started, take this readiness quiz.

1. Draw a model of the fraction $\frac{3}{4}$.
   If you missed this problem, review [link].

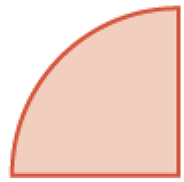
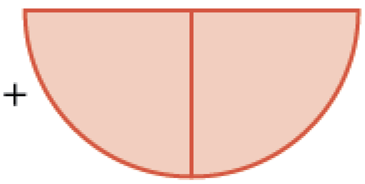
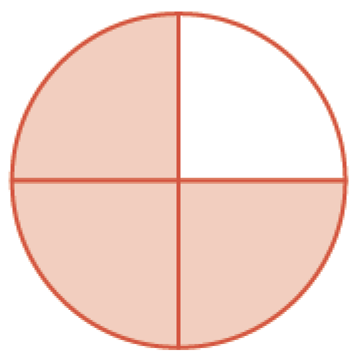
Model Fraction Addition

How many quarters are pictured? One quarter plus 2 quarters equals 3 quarters.

Remember, quarters are really fractions of a dollar. Quarters are another way to say fourths. So the picture of the coins shows that
Equation:

$$\underset{\text{one quarter}}{\frac{1}{4}} + \underset{\text{two quarters}}{\frac{2}{4}} = \underset{\text{three quarters}}{\frac{3}{4}}$$

Let’s use fraction circles to model the same example, $\frac{1}{4} + \frac{2}{4}$.

Start with one $\frac{1}{4}$ piece.		$\frac{1}{4}$
Add two more $\frac{1}{4}$ pieces.	+ 	$+\ \frac{2}{4}$
The result is $\frac{3}{4}$.		$\frac{3}{4}$

So again, we see that

Equation:

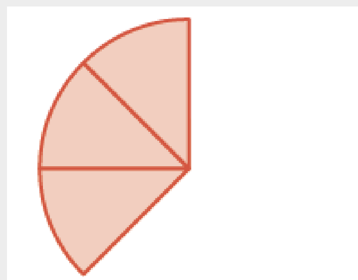
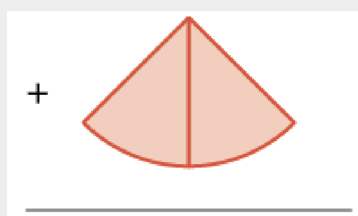
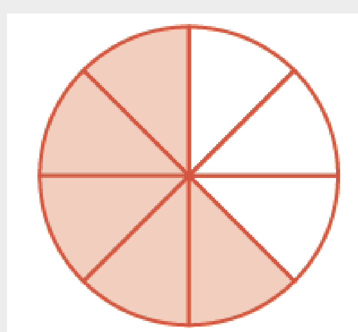
$$\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$$

Example:

Exercise:

Problem: Use a model to find the sum $\frac{3}{8} + \frac{2}{8}$.

Solution:
Solution

Start with three $\frac{1}{8}$ pieces.		$\frac{3}{8}$
Add two $\frac{1}{8}$ pieces.		$+\ \frac{2}{8}$
How many $\frac{1}{8}$ pieces are there?		$\frac{5}{8}$

There are five $\frac{1}{8}$ pieces, or five-eighths. The model shows that $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$.

Note:
Exercise:

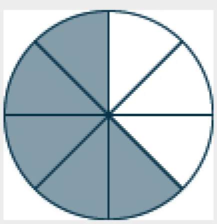
Problem:

Use a model to find each sum. Show a diagram to illustrate your model.

$\frac{1}{8} + \frac{4}{8}$

Solution:

$\frac{5}{8}$



Note:
Exercise:

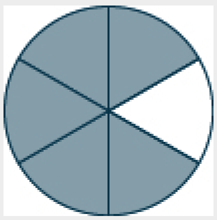
Problem:

Use a model to find each sum. Show a diagram to illustrate your model.

$\frac{1}{6} + \frac{4}{6}$

Solution:

$\frac{5}{6}$



Add Fractions with a Common Denominator

[link] shows that to add the same-size pieces—meaning that the fractions have the same denominator—we just add the number of pieces.

Note:
Fraction Addition
If $a, b,$ and c are numbers where $c \neq 0$, then
Equation:

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

To add fractions with a common denominators, add the numerators and place the sum over the common denominator.

Example:
Exercise:

Problem: Find the sum: $\frac{3}{5} + \frac{1}{5}$.

Solution:
Solution

	$\frac{3}{5}+\frac{1}{5}$
Add the numerators and place the sum over the common denominator.	$\frac{3+1}{5}$
Simplify.	$\frac{4}{5}$

Note:
Exercise:

Problem: Find each sum: $\frac{3}{6}+\frac{2}{6}$.

Solution:

$\frac{5}{6}$

Note:
Exercise:

Problem: Find each sum: $\frac{3}{10}+\frac{7}{10}$.

Solution:

1

Example:
Exercise:

Problem: Find the sum: $\frac{3}{12} + \frac{5}{12}$.

Solution:
Solution

	$\frac{3}{12} + \frac{5}{12}$
Add the numerators and place the sum over the common denominator.	$\frac{3+5}{12}$
Add.	$\frac{8}{12}$
Simplify the fraction.	$\frac{2}{3}$

Note:
Exercise:

Problem: Find each sum: $\frac{2}{15} + \frac{7}{15}$.

Solution:

$\frac{3}{5}$

Model Fraction Subtraction

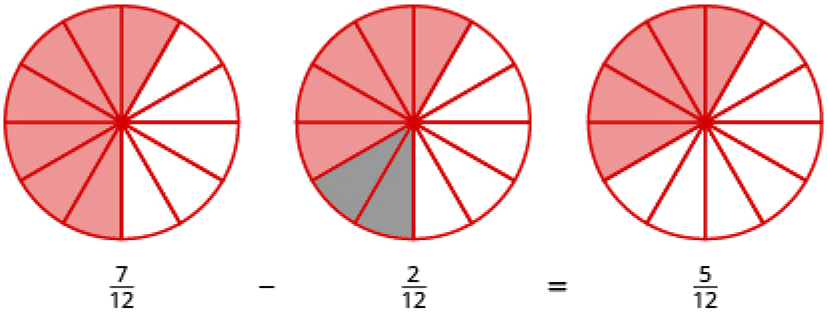
Subtracting two fractions with common denominators is much like adding fractions. Think of a pizza that was cut into 12 slices. Suppose five pieces are eaten for dinner. This means that, after dinner, there are seven pieces (or $\frac{7}{12}$ of the pizza) left in the box. If Leonardo eats 2 of these remaining pieces (or $\frac{2}{12}$ of the pizza), how much is left? There would be 5 pieces left (or $\frac{5}{12}$ of the pizza).

Equation:

$$\frac{7}{12} - \frac{2}{12} = \frac{5}{12}$$

Let's use fraction circles to model the same example, $\frac{7}{12} - \frac{2}{12}$.

Start with seven $\frac{1}{12}$ pieces. Take away two $\frac{1}{12}$ pieces. How many twelfths are left?



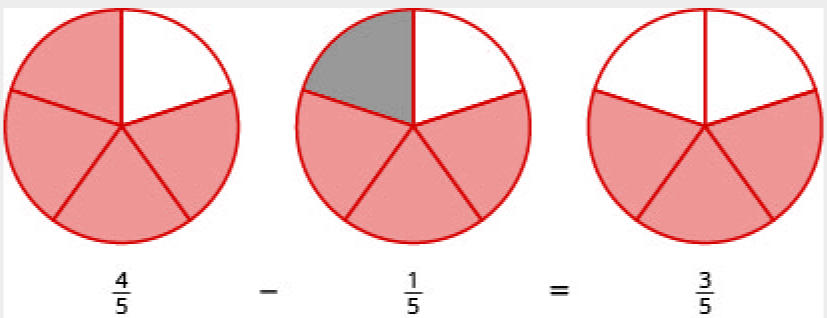
Again, we have five twelfths, $\frac{5}{12}$.

Example:
Exercise:

Problem: Use fraction circles to find the difference: $\frac{4}{5} - \frac{1}{5}$.

Solution:
Solution

Start with four $\frac{1}{5}$ pieces. Take away one $\frac{1}{5}$ piece. Count how many fifths are left. There are three $\frac{1}{5}$ pieces left.



Note:
Exercise:

Problem:

Use a model to find each difference. Show a diagram to illustrate your model.

$\frac{7}{8} - \frac{4}{8}$

Solution:

$\frac{3}{8}$, models may differ.

Note:
Exercise:

Problem:

Use a model to find each difference. Show a diagram to illustrate your model.

$\frac{5}{6} - \frac{4}{6}$

Solution:

$\frac{1}{6}$, models may differ

Subtract Fractions with a Common Denominator

We subtract fractions with a common denominator in much the same way as we add fractions with a common denominator.

Note:
Fraction Subtraction
If $a, b,$ and c are numbers where $c \neq 0$, then
Equation:

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

To subtract fractions with a common denominators, we subtract the numerators and place the difference over the common denominator.

Example:
Exercise:

Problem: Find the difference: $\frac{23}{24} - \frac{14}{24}$.

Solution:
Solution

	$\frac{23}{24} - \frac{14}{24}$
Subtract the numerators and place the difference over the common denominator.	$\frac{23-14}{24}$
Simplify the numerator.	$\frac{9}{24}$
Simplify the fraction by removing common factors.	$\frac{3}{8}$

Note:
Exercise:

Problem: Find the difference: $\frac{19}{28} - \frac{7}{28}$.

Solution:

$\frac{3}{7}$

Note:
Exercise:

Problem: Find the difference: $\frac{27}{32} - \frac{11}{32}$.

Solution:

$\frac{1}{2}$

Now lets do an example that involves both addition and subtraction.

Example:
Exercise:

Problem: Simplify: $\frac{5}{8} - \frac{3}{8} - \frac{1}{8}$.

Solution:
Solution

	$\frac{5}{8} - \frac{3}{8} - \frac{1}{8}$
Combine the numerators over the common denominator.	$\frac{5-3-1}{8}$
Simplify the numerator, working left to right.	$\frac{2-1}{8}$
Subtract the terms in the numerator.	$\frac{1}{8}$

Note:
Exercise:

Problem: Simplify: $\frac{4}{5} - \frac{2}{5} + \frac{3}{5}$.

Solution:

1

Note:
Exercise:

Problem: Simplify: $\frac{5}{9} - \frac{4}{9} + \frac{5}{9}$.

Solution:

$\frac{2}{3}$

Note:
ACCESS ADDITIONAL ONLINE RESOURCES

- Adding Fractions With Pattern Blocks
- Adding Fractions With Like Denominators
- Subtracting Fractions With Like Denominators

Key Concepts

- **Fraction Addition**
  - If a, b, and c are numbers where $c \neq 0$, then $\frac{a}{c} + \frac{b}{c} = \frac{a+c}{c}$.
  - To add fractions, add the numerators and place the sum over the common denominator.
- **Fraction Subtraction**
  - If a, b, and c are numbers where $c \neq 0$, then $\frac{a}{c} - \frac{b}{c} = \frac{a-c}{c}$.
  - To subtract fractions, subtract the numerators and place the difference over the common denominator.

Exercises

Practice Makes Perfect

Model Fraction Addition

In the following exercises, use a model to add the fractions. Show a diagram to illustrate your model.

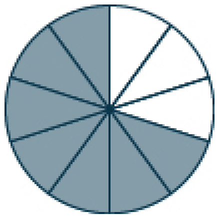
Exercise:

Problem: $\frac{2}{5} + \frac{1}{5}$

Exercise:

Problem: $\frac{3}{10} + \frac{4}{10}$

Solution:



$\frac{7}{10}$

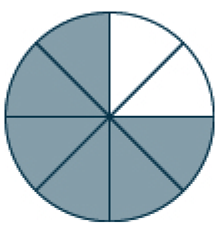
Exercise:

Problem: $\frac{1}{6} + \frac{3}{6}$

Exercise:

Problem: $\frac{3}{8} + \frac{3}{8}$

Solution:



$\frac{3}{4}$

Add Fractions with a Common Denominator

In the following exercises, find each sum.

Exercise:

Problem: $\frac{4}{9} + \frac{1}{9}$

Exercise:

Problem: $\frac{2}{9} + \frac{5}{9}$

Solution:

$\frac{7}{9}$

Exercise:

Problem: $\frac{6}{13} + \frac{7}{13}$

Exercise:

Problem: $\frac{9}{15} + \frac{7}{15}$

Solution:

$\frac{16}{15}$

Exercise:

Problem: $\frac{1}{8} + \frac{5}{8}$

Solution:

$\frac{3}{4}$

Exercise:

Problem: $\frac{5}{16} + \frac{9}{16}$

Solution:

$\frac{7}{8}$

Exercise:

Problem: $\frac{17}{19} - \frac{9}{19}$

Solution:

$\frac{8}{19}$

Exercise:

Problem: $\frac{5}{12} + \frac{2}{12} + \frac{3}{12}$

Solution:

$\frac{5}{6}$

Model Fraction Subtraction

In the following exercises, use a model to subtract the fractions. Show a diagram to illustrate your model.

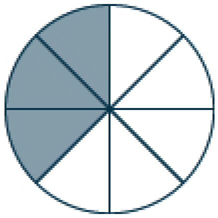
Exercise:

Problem: $\frac{5}{8} - \frac{2}{8}$

Exercise:

Problem: $\frac{5}{6} - \frac{2}{6}$

Solution:



$\frac{1}{2}$

Subtract Fractions with a Common Denominator

In the following exercises, find the difference.

Exercise:

Problem: $\frac{4}{5} - \frac{1}{5}$

Exercise:

Problem: $\frac{4}{5} - \frac{3}{5}$

Solution:

$\frac{1}{5}$

Exercise:

Problem: $\frac{11}{15} - \frac{7}{15}$

Exercise:

Problem: $\frac{9}{13} - \frac{4}{13}$

Solution:

$\frac{5}{13}$

Exercise:

Problem: $\frac{11}{12} - \frac{5}{12}$

Exercise:

Problem: $\frac{7}{12} - \frac{5}{12}$

Solution:

$\frac{1}{6}$

Exercise:

Problem: $\frac{13}{7} - \frac{11}{7}$

Solution:

$\frac{2}{7}$

Exercise:

Problem: $\frac{8}{11} - \frac{5}{11}$

Solution:

$\frac{3}{11}$

Everyday Math

Exercise:

Problem:

Trail Mix Jacob is mixing together nuts and raisins to make trail mix. He has $\frac{6}{10}$ of a pound of nuts and $\frac{3}{10}$ of a pound of raisins. How much trail mix will that make?

Exercise:

Problem:

Baking Janet needs $\frac{5}{8}$ of a cup of flour for a recipe she is making. She only has $\frac{3}{8}$ of a cup of flour and will ask to borrow the rest from her next-door neighbor. How much flour does she have to borrow?

Solution:

$\frac{1}{4}$ cup

Writing Exercises

Exercise:

Problem:

Greg dropped his case of drill bits and three of the bits fell out. The case has slots for the drill bits, and the slots are arranged in order from smallest to largest. Greg needs to put the bits that fell out back in the case in the empty slots. Where do the three bits go? Explain how you know.
Bits in case: $\frac{1}{16}, \frac{1}{8}, \_\_\_, \_\_\_, \frac{5}{16}, \frac{3}{8}, \_\_\_, \frac{1}{2}, \frac{9}{16}, \frac{5}{8}$.
Bits that fell out: $\frac{7}{16}, \frac{3}{16}, \frac{1}{4}$.

Exercise:

Problem:

After a party, Lupe has $\frac{5}{12}$ of a cheese pizza, $\frac{4}{12}$ of a pepperoni pizza, and $\frac{4}{12}$ of a veggie pizza left. Will all the slices fit into 1 pizza box? Explain your reasoning.

Solution:

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can…	Confidently	With some help	No-I don't get it!
model fraction addition.			
add fractions with a common denominator.			
model fraction subtraction.			
subtract fractions with a common denominator.			
find the least common denominator (LCD).			
convert fractions to equivalent fractions with the LCD.			

ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Add and Subtract Fractions with Different Denominators Beginning Level
By the end of this section, you will be able to:

- Find the least common denominator (LCD)
- Convert fractions to equivalent fractions with the LCD
- Add and subtract fractions with different denominators

Note:
Before you get started, take this readiness quiz.

1. Find two fractions equivalent to $\frac{5}{6}$.
   If you missed this problem, review [link].
2. Simplify: $\frac{5}{8} - \frac{3}{8} - \frac{1}{8}$.
   If you missed this problem, review [link].

Find the Least Common Denominator

In the previous section, we explained how to add and subtract fractions with a common denominator. But how can we add and subtract fractions with unlike denominators?

Let's think about coins again. Can you add one quarter and one dime? You could say there are two coins, but that's not very useful. To find the total value of one quarter plus one dime, you change them to the same kind of unit—cents. One quarter equals 25 cents and one dime equals 10 cents, so the sum is 35 cents. See [link].

Together, a quarter and a dime are worth 35 cents, or $\frac{35}{100}$ of a dollar.

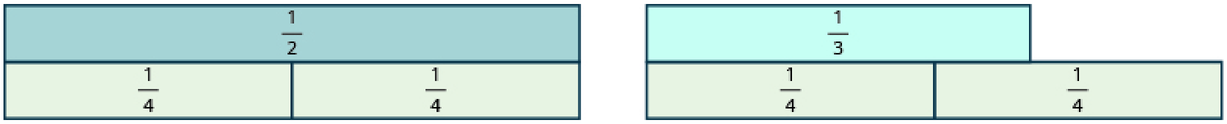
Similarly, when we add fractions with different denominators we have to convert them to equivalent fractions with a common denominator. With the coins, when we convert to cents, the denominator is 100. Since there are 100 cents in one dollar, 25 cents is $\frac{25}{100}$ and 10 cents is $\frac{10}{100}$. So we add $\frac{25}{100} + \frac{10}{100}$ to get $\frac{35}{100}$, which is 35 cents.

You have practiced adding and subtracting fractions with common denominators. Now let’s see what you need to do with fractions that have different denominators.

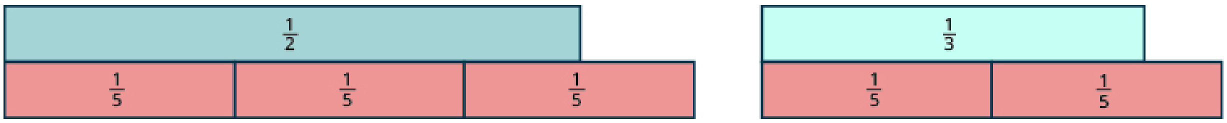
First, we will use fraction towers to model finding the common denominator of $\frac{1}{2}$ and $\frac{1}{3}$.

We’ll start with one $\frac{1}{2}$ tower and $\frac{1}{3}$ tower. We want to find a common fraction tower that we can use to match *both* $\frac{1}{2}$ and $\frac{1}{3}$ exactly.

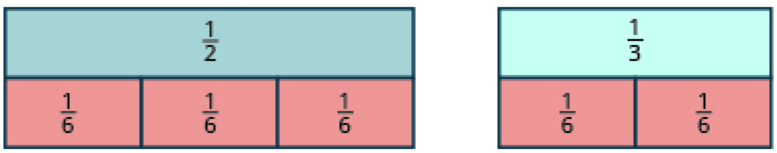
If we try the $\frac{1}{4}$ pieces, 2 of them exactly match the $\frac{1}{2}$ piece, but they do not exactly match the $\frac{1}{3}$ piece.



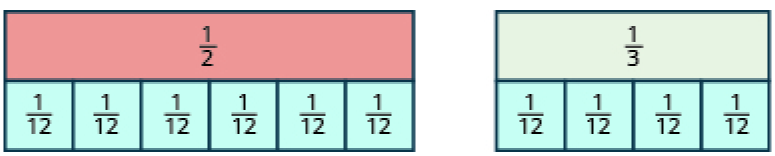
If we try the $\frac{1}{5}$ pieces, they do not exactly match the $\frac{1}{2}$ piece or the $\frac{1}{3}$ piece.



If we try the $\frac{1}{6}$ pieces, we see that exactly 3 of them match the $\frac{1}{2}$ piece, and exactly 2 of them match the $\frac{1}{3}$ piece.



If we were to try the $\frac{1}{12}$ pieces, they would also work.



Even smaller pieces, such as $\frac{1}{24}$ and $\frac{1}{48}$, would also exactly match the $\frac{1}{2}$ piece and the $\frac{1}{3}$ piece.

The denominator of the largest piece that matches both fractions is the **least common denominator (LCD)** of the two fractions. Recall that for the same numerator, the smaller the denominator then the larger the piece will be.

The least common denominator of $\frac{1}{2}$ and $\frac{1}{3}$ is 6.

Notice that all of the pieces that exactly match $\frac{1}{2}$ and $\frac{1}{3}$ have something in common: Their denominators are common multiples of 2 and 3, the denominators of $\frac{1}{2}$ and $\frac{1}{3}$.

The least common multiple (LCM) of the denominators is 6, and so we say that 6 is the least common denominator (LCD) of the fractions $\frac{1}{2}$ and $\frac{1}{3}$.

Note:
Least Common Denominator
The **least common denominator (LCD)** of two fractions is the least common multiple (LCM) of their denominators.

To find the LCD of two fractions, we will find the LCM of their denominators. One procedure to find the LCM is shown below. We only use the denominators of the fractions, not the numerators, when finding the LCD.

Example:
Exercise:

Problem: Find the LCD for the fractions $\frac{5}{6}$ and $\frac{3}{10}$.

Solution:
Solution

Start a list of multiples for each denominator.	6, 12, 18, 24, 30, 36 10, 20, 30, 40

Scan the lists to see the smallest value they have in common.	30
The LCM of 6 and 10 is 30, so the LCD of $\frac{5}{6}$ and $\frac{3}{10}$ is 30.	LCD of $\frac{5}{6}$ and $\frac{3}{10}$ is 30.

Note:
Exercise:

Problem:

Find the least common denominator for the fractions: $\frac{7}{12}$ and $\frac{11}{15}$.

Solution:

60

Note:
Exercise:

Problem:

Find the least common denominator for the fractions: $\frac{13}{15}$ and $\frac{17}{5}$.

Solution:

15

To find the LCD of two fractions, find the LCM of their denominators.

Note:
Find the least common denominator (LCD) of two fractions.

Start a list of multiples for each denominator.
Scan the list starting at the low end to see if they have a common multiple.
The first common multiple found is the LCM.
If no common multiple is found increase the list with the smallest last value.
Check to see if the new value is a common multiple.
If not repeat adding a value to the list with the smallest value and check again.

Example:
Exercise:

Problem:

Find the least common denominator for the fractions $\frac{8}{15}$ and $\frac{11}{12}$.

Solution:
Solution

To find the LCD, we find the LCM of the denominators.

Find the LCM of 15 and 12.

15, 30, 45
12, 24, 36
No common multiple so far

15, 30, 45
12, 24, 36, 48
No common multiple so far

15, 30, 45, 60

12, 24, 36, 48
No common multiple so far

15, 30, 45, 60
12, 24, 36, 48, 60

LCM = 60.

The LCM of 15 and 12 is 60. So, the LCD of $\frac{8}{15}$ and $\frac{11}{12}$ is 60.

Note:
Exercise:

Problem:

Find the least common denominator for the fractions: $\frac{3}{12}$ and $\frac{5}{8}$.

Solution:

24

Note:
Exercise:

Problem:

Find the least common denominator for the fractions: $\frac{9}{14}$ and $\frac{21}{10}$.

Solution:

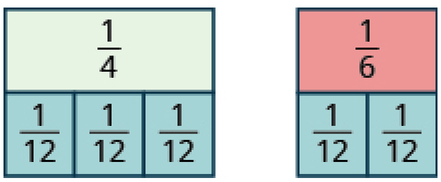
70

Convert Fractions to Equivalent Fractions with the LCD

Earlier, we used fraction tiles to see that the LCD of $\frac{1}{4}$ and $\frac{1}{6}$ is 12. We saw that three $\frac{1}{12}$ pieces exactly matched $\frac{1}{4}$ and two $\frac{1}{12}$ pieces exactly matched $\frac{1}{6}$, so

Equation:

$$\frac{1}{4} = \frac{3}{12}\text{and}\frac{1}{6} = \frac{2}{12}.$$



We say that $\frac{1}{4}$ and $\frac{3}{12}$ are equivalent fractions and also that $\frac{1}{6}$ and $\frac{2}{12}$ are equivalent fractions.

We can use the Equivalent Fractions Property to algebraically change a fraction to an equivalent one. Remember, two fractions are equivalent if they have the same value. The Equivalent Fractions Property is repeated below for reference.

Note:
Equivalent Fractions Property
If a, b, c are whole numbers where $b \neq 0, c \neq 0,$ then
Equation:

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c} \quad \text{and} \quad \frac{a \cdot c}{b \cdot c} = \frac{a}{b}$$

To add or subtract fractions with different denominators, we will first have to convert each fraction to an equivalent fraction with the LCD. Let's see how to change $\frac{1}{4}$ and $\frac{1}{6}$ to equivalent fractions with denominator 12 without using models.

Example:
Exercise:

Problem:

Convert $\frac{1}{4}$ and $\frac{1}{6}$ to equivalent fractions with denominator 12, their LCD.

Solution:
Solution

Find the LCD.	The LCD of $\frac{1}{4}$ and $\frac{1}{6}$ is 12.
Find the number to multiply 4 to get 12.	$4 \cdot 3 = 12$
Find the number to multiply 6 to get 12.	$6 \cdot 2 = 12$
Use the Equivalent Fractions Property to convert each fraction to an equivalent fraction with the LCD,	$\frac{1}{4}$ $\frac{1}{6}$ $\frac{1 \cdot 3}{4 \cdot 3}$ $\frac{1 \cdot 2}{6 \cdot 2}$

multiplying both the numerator and denominator of each fraction by the same number.	
Simplify the numerators and denominators.	$\frac{3}{12}$ $\frac{2}{12}$

We do not reduce the resulting fractions. If we did, we would get back to our original fractions and lose the common denominator.

Note:
Exercise:

Problem: Change to equivalent fractions with the LCD:

$\frac{3}{4}$ and $\frac{5}{6}$, $\text{LCD} = 12$

Solution:

$\frac{9}{12}, \frac{10}{12}$

Note:
Exercise:

Problem: Change to equivalent fractions with the LCD:

$\frac{7}{12}$ and $\frac{11}{15}$, $\text{LCD} = 60$

Solution:

$\frac{35}{60}, \frac{44}{60}$

Note:
Convert two fractions to equivalent fractions with their LCD as the common denominator.

Find the LCD.
For each fraction, determine the number needed to multiply the denominator to get the LCD.
Use the Equivalent Fractions Property to multiply both the numerator and denominator by the number you found in Step 2.
Simplify the numerator and denominator.

Example:
Exercise:

Problem:

Convert $\frac{8}{15}$ and $\frac{11}{24}$ to equivalent fractions with denominator 120, their LCD.

Solution:
Solution

	The LCD is 120. We will start at Step 2.
Find the number that must multiply 15 to get 120.	$15 \cdot 8 = 120$
Find the number that must	$24 \cdot 5 = 120$

multiply 24 to get 120.	
Use the Equivalent Fractions Property.	$\frac{8 \cdot 8}{15 \cdot 8}$ $\frac{11 \cdot 5}{24 \cdot 5}$
Simplify the numerators and denominators.	$\frac{64}{120}$ $\frac{55}{120}$

Note:
Exercise:

Problem: Change to equivalent fractions with the LCD:

$\frac{13}{24}$ and $\frac{17}{32}$, LCD 96

Solution:

$\frac{52}{96}$, $\frac{51}{96}$

Note:
Exercise:

Problem: Change to equivalent fractions with the LCD:

$\frac{9}{28}$ and $\frac{27}{32}$, LCD 224

Solution:

$\frac{72}{224}, \frac{189}{224}$

Add and Subtract Fractions with Different Denominators

Once we have converted two fractions to equivalent forms with common denominators, we can add or subtract them by adding or subtracting the numerators.

Note:
Add or subtract fractions with different denominators.

Find the LCD.
Convert each fraction to an equivalent form with the LCD as the denominator.
Add or subtract the fractions.
Write the result in simplified form.

Example:
Exercise:

Problem: Add: $\frac{1}{2} + \frac{1}{3}$.

Solution:
Solution

	$\frac{1}{2}+\frac{1}{3}$
Find the LCD of 2, 3. 2, 4, 6 3, 6	LCD = 6
Change into equivalent fractions with the LCD 6.	$\frac{1 \cdot 3}{2 \cdot 3}+\frac{1 \cdot 2}{3 \cdot 2}$
Simplify the numerators and denominators.	$\frac{3}{6}+\frac{2}{6}$
Add.	$\frac{5}{6}$

Remember, always check to see if the answer can be simplified. Since 5 and 6 have no common factors, the fraction $\frac{5}{6}$ cannot be reduced.

You may want to do this problem with Fraction Towers to verify that the answer is correct.

Note:
Exercise:

Problem: Add: $\frac{1}{4}+\frac{1}{3}$.

Solution:

$\frac{7}{12}$

Note:
Exercise:

Problem: Add: $\frac{1}{2}+\frac{1}{5}$.

Solution:

$\frac{7}{10}$

Example:
Exercise:

Problem: Subtract: $\frac{1}{2}-\frac{1}{4}$.

Solution:
Solution

	$\frac{1}{2}-\frac{1}{4}$
Find the LCD of 2 and 4. 2, 4 4	LCD = 4
Rewrite as equivalent fractions using the LCD 4.	$\frac{1\cdot 2}{2\cdot 2}-\left(\frac{1}{4}\right)$

Simplify the first fraction.	$\frac{2}{4} - \frac{1}{4}$
Subtract.	$\frac{2-1}{4}$
Simplify.	$\frac{1}{4}$

One of the fractions already had the least common denominator, so we only had to convert the other fraction.

Note:
Exercise:

Problem: Simplify: $\frac{1}{2} - \frac{1}{8}$.

Solution:

$\frac{3}{8}$

Note:
Exercise:

Problem: Simplify: $\frac{1}{3} - \frac{1}{6}$.

Solution:

$\frac{1}{6}$

Note:

Exercise:

Problem: Add: $\frac{7}{12}+\frac{11}{15}$.

Solution:

$\frac{79}{60}$

Note:
Exercise:

Problem: Add: $\frac{13}{15}+\frac{17}{20}$.

Solution:

$\frac{103}{60}$

Note:
Exercise:

Problem: Subtract: $\frac{13}{24}-\frac{17}{32}$.

Solution:

$\frac{1}{96}$

Note:
Exercise:

Problem: Subtract: $\frac{21}{32} - \frac{9}{28}$.

Solution:

$\frac{75}{224}$

Key Concepts

- **Find the least common denominator (LCD) of two fractions.**

  Start a list of multiples for each denominator.
  Scan the list starting at the low end to see if they have a common multiple.
  The first common multiple found is the LCM.
  If no common multiple is found increase the list with the smallest last value.
  Check to see if the new value is a common multiple.
  If not repeat adding a value to the list with the smallest value and check again.

- **Equivalent Fractions Property**

  - If $a, b,$ and c are whole numbers where $b \neq 0$, $c \neq 0$ then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$

- **Convert two fractions to equivalent fractions with their LCD as the common denominator.**

  Find the LCD.
  For each fraction, determine the number needed to multiply the denominator to get the LCD.
  Use the Equivalent Fractions Property to multiply both the numerator and denominator by the number you found in Step 2.
  Simplify the numerator and denominator.

- **Add or subtract fractions with different denominators.**

  Find the LCD.
  Convert each fraction to an equivalent form with the LCD as the denominator.
  Add or subtract the fractions.
  Write the result in simplified form.

- **Summary of Fraction Operations**

  - **Fraction addition:** Add the numerators and place the sum over the common denominator. If the fractions have different denominators, first convert them to equivalent forms with the LCD.
    Equation:

    $$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

  - **Fraction subtraction:** Subtract the numerators and place the difference over the common denominator. If the fractions have different denominators, first convert them to equivalent forms with the LCD.
    Equation:

    $$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

Exercises

Practice Makes Perfect

Find the Least Common Denominator (LCD)

In the following exercises, find the least common denominator (LCD) for each set of fractions.
Exercise:

Problem: $\frac{2}{3}$ and $\frac{3}{4}$

Exercise:

Problem: $\frac{3}{4}$ and $\frac{2}{5}$

Solution:

20

Exercise:

Problem: $\frac{7}{12}$ and $\frac{5}{8}$

Exercise:

Problem: $\frac{9}{16}$ and $\frac{7}{12}$

Solution:

48

Exercise:

Problem: $\frac{13}{30}$ and $\frac{25}{42}$

Exercise:

Problem: $\frac{23}{30}$ and $\frac{5}{48}$

Solution:

240

Exercise:

Problem: $\frac{21}{35}$ and $\frac{39}{56}$

Exercise:

Problem: $\frac{18}{35}$ and $\frac{33}{49}$

Solution:

245

Exercise:

Problem: $\frac{2}{3}$, $\frac{1}{6}$, and $\frac{3}{4}$

Exercise:

Problem: $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{3}{5}$

Solution:

60

Convert Fractions to Equivalent Fractions with the LCD

In the following exercises, convert to equivalent fractions using the LCD.

Exercise:

Problem: $\frac{1}{3}$ and $\frac{1}{4}$, $\text{LCD} = 12$

Exercise:

Problem: $\frac{1}{4}$ and $\frac{1}{5}$, $\text{LCD} = 20$

Solution:

$\frac{5}{20}, \frac{4}{20}$

Exercise:

Problem: $\frac{5}{12}$ and $\frac{7}{8}$, $\text{LCD} = 24$

Exercise:

Problem: $\frac{7}{12}$ and $\frac{5}{8}$, $\text{LCD} = 24$

Solution:

$\frac{14}{24}, \frac{15}{24}$

Exercise:

Problem: $\frac{13}{16}$ and $-\frac{11}{12}$, $\text{LCD} = 48$

Exercise:

Problem: $\frac{11}{16}$ and $-\frac{5}{12}$, $\text{LCD} = 48$

Solution:

$\frac{33}{48}, -\frac{20}{48}$

Exercise:

Problem: $\frac{1}{3}, \frac{5}{6}$, and $\frac{3}{4}$, $\text{LCD} = 12$

Exercise:

Problem: $\frac{1}{3}, \frac{3}{4}$, and $\frac{3}{5}$, $\text{LCD} = 60$

Solution:

$\frac{20}{60}, \frac{45}{60}, \frac{36}{60}$

Add and Subtract Fractions with Different Denominators

In the following exercises, add or subtract. Write the result in simplified form.

Exercise:

Problem: $\frac{1}{3}+\frac{1}{5}$

Exercise:

Problem: $\frac{1}{4}+\frac{1}{5}$

Solution:

$\frac{9}{20}$

Exercise:

Problem: $\frac{1}{2}+\frac{1}{7}$

Exercise:

Problem: $\frac{1}{3}+\frac{1}{8}$

Solution:

$\frac{11}{24}$

Exercise:

Problem: $\frac{2}{3}+\frac{3}{4}$

Exercise:

Problem: $\frac{3}{4}+\frac{2}{5}$

Solution:

$\frac{23}{20}$

Exercise:

Problem: $\frac{7}{12}+\frac{5}{8}$

Exercise:

Problem: $\frac{5}{12}+\frac{3}{8}$

Solution:

$\frac{19}{24}$

Exercise:

Problem: $\frac{7}{12}-\frac{9}{16}$

Exercise:

Problem: $\frac{7}{16}-\frac{5}{12}$

Solution:

$\frac{1}{48}$

Exercise:

Problem: $\frac{11}{12}-\frac{3}{8}$

Exercise:

Problem: $\frac{5}{8}-\frac{7}{12}$

Solution:

$\frac{1}{24}$

Exercise:

Problem: $\frac{2}{3} - \frac{3}{8}$

Exercise:

Problem: $\frac{5}{6} - \frac{3}{4}$

Solution:

$\frac{1}{12}$

Exercise:

Problem: $1 + \frac{7}{8}$

Exercise:

Problem: $1 + \frac{5}{6}$

Solution:

$\frac{11}{6}$

Exercise:

Problem: $1 - \frac{5}{9}$

Exercise:

Problem: $1 - \frac{3}{10}$

Solution:

$\frac{7}{10}$

Everyday Math

Exercise:

Problem:

Decorating Laronda is making covers for the throw pillows on her sofa. For each pillow cover, she needs $\frac{3}{16}$ yard of print fabric and $\frac{3}{8}$ yard of solid fabric. What is the total amount of fabric Laronda needs for each pillow cover?

Exercise:

Problem:

Baking Vanessa is baking chocolate chip cookies and oatmeal cookies. She needs $1\frac{1}{4}$ cups of sugar for the chocolate chip cookies, and $1\frac{1}{8}$ cups for the oatmeal cookies How much sugar does she need altogether?

Solution:

She needs $2\frac{3}{8}$ cups

Writing Exercises

Exercise:

Problem:

Explain why it is necessary to have a common denominator to add or subtract fractions.

Exercise:

Problem: Explain how to find the LCD of two fractions.

Solution:

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can…	Confidently	With some help	No-I don't get it!
add and subtract fractions with different denominators.			
identify and use fraction operations.			
use the order of operations to simplify complex fractions.			
evaluate variable expressions with fractions.			

ⓑ After looking at the checklist, do you think you are well prepared for the next section? Why or why not?

Glossary

least common denominator (LCD)
: The least common denominator (LCD) of two fractions is the least common multiple (LCM) of their denominators.

Multiply and Divide Fractions Beginning Level
By the end of this section, you will be able to:

- Multiply fractions
- Find reciprocals
- Divide fractions

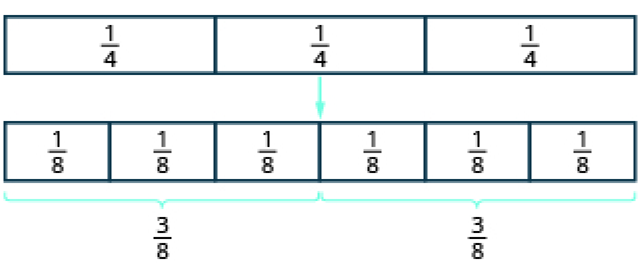
Note:
Before you get started, take this readiness quiz.

1. Draw a model of the fraction $\frac{3}{4}$.
   If you missed this problem, review [link].
2. Find two fractions equivalent to $\frac{5}{6}$.
   Answers may vary. Acceptable answers include $\frac{10}{12}$, $\frac{15}{18}$, $\frac{50}{60}$, etc.
   If you missed this problem, review [link].

Multiply fractions

A model may help you understand multiplication of fractions. We will use fraction tiles to model $\frac{1}{2} \cdot \frac{3}{4}$. To multiply $\frac{1}{2}$ and $\frac{3}{4}$, think $\frac{1}{2}$ of $\frac{3}{4}$.

Start with fraction tiles for three-fourths. To find one-half of three-fourths, we need to divide them into two equal groups. Since we cannot divide the three $\frac{1}{4}$ tiles evenly into two parts, we exchange them for smaller tiles.



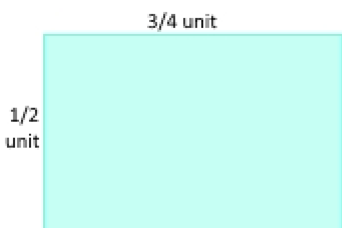
We see $\frac{6}{8}$ is equivalent to $\frac{3}{4}$. Taking half of the six $\frac{1}{8}$ tiles gives us three $\frac{1}{8}$ tiles, which is $\frac{3}{8}$.

Therefore,

Equation:

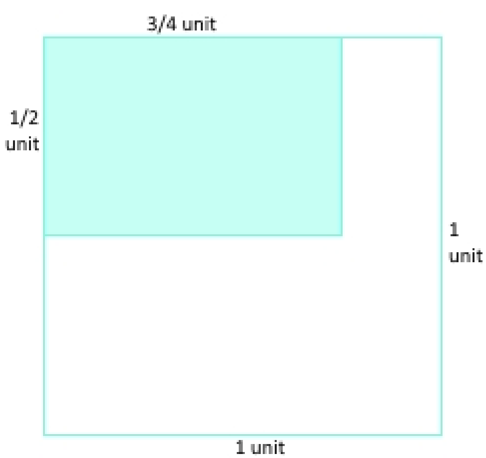
$$\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$$

The area model of multiplication can also be used to model this problem. We need a rectangle with one side $\frac{1}{2}$ of a unit long and the other side $\frac{3}{4}$ of a unit long.

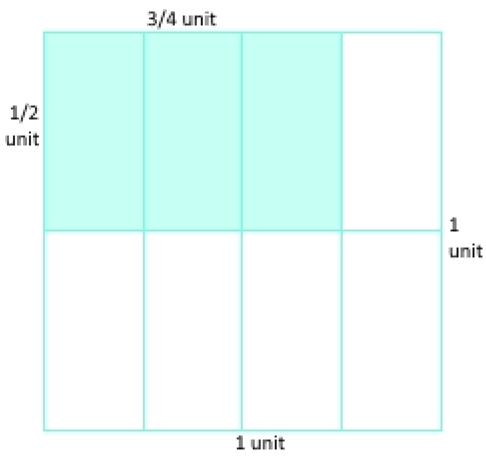


The area is the product of the sides.

We can find the area by extending the sides of this rectangle to make a unit square.



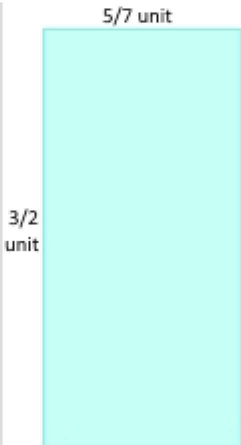
Draw in the lines that show the unit rectangle divided horizontally into half ($\frac{1}{2}$) and vertically into fourths ($\frac{1}{4}$).



All of the small rectangles are the same size. How many of them make up the unit square?
There are $2 \cdot 4 = 8$ of them; each one is $\frac{1}{8}$ of the unit square.

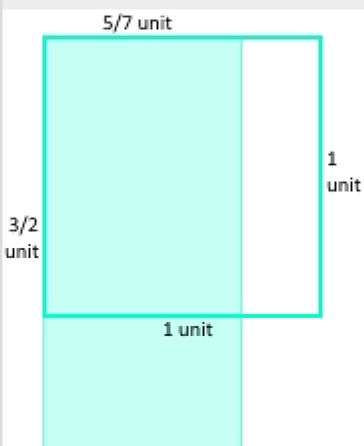
How many are shaded?
$1 \cdot 3 = 3$ of them. Therefore the area is $\frac{3}{8}$ of the square unit.

Note:
The area model of multiplication can also be used to model this problem. We need a rectangle with one side $\frac{3}{2}$ units long and the other side $\frac{5}{7}$ of a unit long.

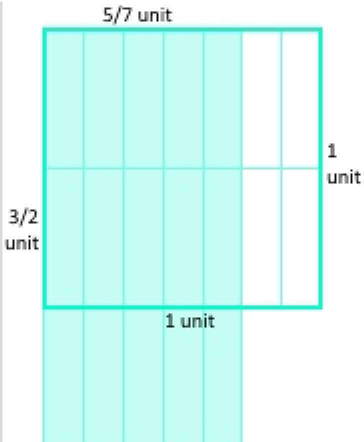


The area is the product of the sides.

We can find the area by extending the sides of this rectangle to make a unit square. The height is more than 1 unit long, so it is not fully included in the unit square.



Draw in the lines that show the unit rectangle divided horizontally into halves ($\frac{1}{2}$) and vertically into sevenths ($\frac{1}{7}$).



All of the small rectangles are the same size. How many of them make up the unit square?
There are $2 \cdot 7 = 14$ of them; each one is $\frac{1}{14}$ of the unit square.

How many are shaded?
$3 \cdot 5 = 15$ of them. Therefore the area is $\frac{15}{14}$ of the square unit. It's OK that some of these rectangles are outside of the unit square.
Exercise:

Problem: Use a diagram to model: $\frac{3}{4} \cdot \frac{5}{7}$.

Solution:

$\frac{15}{28}$

In the examples and exercises you may have noticed that we multiply the numerators of the factors to get the numerator of the product and multiply the denominators of the factors to get the denominator of the product. Normally, we then write the fraction in simplified form.

Note:
Fraction Multiplication
If $a, b, c,$ and d are numbers where $b \neq 0$ and $d \neq 0$, then
Equation:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Example:
Exercise:

Problem: Multiply, and write the answer in simplified form: $\frac{3}{4} \cdot \frac{1}{5}$.

Solution:
Solution

	$\frac{3}{4} \cdot \frac{1}{5}$
Multiply the numerators; multiply the denominators.	$\frac{3 \cdot 1}{4 \cdot 5}$
Simplify.	$\frac{3}{20}$

There are no common factors, so the fraction is simplified.

Note:
Exercise:

Problem: Multiply, and write the answer in simplified form: $\frac{1}{3} \cdot \frac{2}{5}$.

Solution:

$\frac{2}{15}$

Note:
Exercise:

Problem: Multiply, and write the answer in simplified form: $\frac{3}{5} \cdot \frac{7}{8}$.

Solution:

$\frac{21}{40}$

When multiplying fractions, the properties of positive and negative numbers still apply. It is a good idea to determine the sign of the product as the first step. In Example 4.26 we will multiply two negatives, so the product will be positive.

Example:
Exercise:

Problem: Multiply, and write the answer in simplified form: $\frac{5}{8}\left(\frac{2}{3}\right)$.

Solution:
Solution

	$\frac{5}{8}\left(\frac{2}{3}\right)$
Multiply the numerators, multiply the denominators.	$\frac{5\cdot 2}{8\cdot 3}$
Simplify.	$\frac{10}{24}$
Look for common factors in the numerator and denominator. Rewrite showing common factors.	$\frac{5\cdot\cancel{2}}{12\cdot\cancel{2}}$
Remove common factors.	$\frac{5}{12}$

Another way to find this product involves removing common factors earlier.

	$\frac{5}{8}\left(\frac{2}{3}\right)$
Multiply.	$\frac{5\cdot 2}{8\cdot 3}$
Show common factors and then remove them.	$\frac{5\cdot\cancel{2}}{4\cdot\cancel{2}\cdot 3}$
Multiply remaining factors.	$\frac{5}{12}$

We get the same result.

Note:
Exercise:

Problem: Multiply, and write the answer in simplified form: $\frac{4}{7}\left(\frac{5}{8}\right)$.

Solution:

$\frac{5}{14}$

Note:
Exercise:

Problem: Multiply, and write the answer in simplified form: $\frac{7}{12}\left(\frac{8}{9}\right)$.

Solution:

$\frac{14}{27}$

Note:
Exercise:

Problem: Multiply, and write the answer in simplified form: $\frac{10}{28} \cdot \frac{8}{15}$.

Solution:

$\frac{4}{21}$

Note:

Exercise:

Problem: Multiply, and write the answer in simplified form: $\frac{9}{20} \cdot \frac{5}{12}$.

Solution:

$\frac{3}{16}$

When multiplying a fraction by an integer, it may be helpful to write the integer as a fraction. Any integer, a, can be written as $\frac{a}{1}$. So, $3 = \frac{3}{1}$, for example.

Example:
Exercise:

Problem: Multiply, and write the answer in simplified form:

$\frac{1}{7} \cdot 56$

Solution:
Solution

	$\frac{1}{7} \cdot 56$
Write 56 as a fraction.	$\frac{1}{7} \cdot \frac{56}{1}$
Determine the sign of the product; multiply.	$\frac{56}{7}$

Simplify.	8

Note:
Exercise:

Problem: Multiply, and write the answer in simplified form:

ⓐ $\frac{1}{8} \cdot 72$
ⓑ $\frac{11}{3}(9)$

Solution:

ⓐ 9
ⓑ 33

Note:
Exercise:

Problem: Multiply, and write the answer in simplified form:

ⓐ $\frac{3}{8} \cdot 64$
ⓑ $16 \cdot \frac{11}{12}$

Solution:

ⓐ 24

ⓑ $\frac{44}{3}$

Find Reciprocals

The fractions $\frac{2}{3}$ and $\frac{3}{2}$ are related to each other in a special way. So are $\frac{10}{7}$ and $\frac{7}{10}$. Do you see how? Besides looking like upside-down versions of one another, if we were to multiply these pairs of fractions, the product would be 1.

Equation:

$$\frac{2}{3} \cdot \frac{3}{2} = 1 \quad \text{and} \quad \frac{10}{7}\left(\frac{7}{10}\right) = 1$$

Such pairs of numbers are called reciprocals.

Note:

Reciprocal

The **reciprocal** of the fraction $\frac{a}{b}$ is $\frac{b}{a}$, where $a \neq 0$ and $b \neq 0$,

A number and its reciprocal have a product of 1. Notice that if X is the reciprocal of Y then Y is the reciprocal of X.

Equation:

$$\frac{a}{b} \cdot \frac{b}{a} = 1$$

To find the reciprocal of a fraction, we invert the fraction. This means that we place the numerator in the denominator and the denominator in the numerator.

The number zero does not have a reciprocal. Why? A number and its reciprocal multiply to 1. Is there any number r so that $0 \cdot r = 1$? No. So, the number 0 does not have a reciprocal.
If we were to invert a fraction with zero in the numerator then new fraction would have zero in the denominator. But division by zero is undefined.

Example:
Exercise:

Problem:

Find the reciprocal of each number. Then check that the product of each number and its reciprocal is 1.

ⓐ $\frac{4}{9}$
ⓑ $\frac{1}{6}$
ⓒ $\frac{14}{5}$
ⓓ 7

Solution:
Solution

To find the reciprocals, we keep the sign and invert the fractions.

ⓐ	
Find the reciprocal of $\frac{4}{9}$.	The reciprocal of $\frac{4}{9}$ is $\frac{9}{4}$.
Check:	

Multiply the number and its reciprocal.	$\frac{4}{9} \cdot \frac{9}{4}$
Multiply numerators and denominators.	$\frac{36}{36}$
Simplify.	$1 \checkmark$

ⓑ	
Find the reciprocal of $\frac{1}{6}$.	$\frac{6}{1}$
Simplify.	6
Check:	$\frac{1}{6} \cdot (6)$
	$1 \checkmark$

ⓒ	
Find the reciprocal of $\frac{14}{5}$.	$\frac{5}{14}$
Check:	$\frac{14}{5} \left(\frac{5}{14}\right)$

	$\frac{70}{70}$
	$1\checkmark$

ⓓ	
Find the reciprocal of 7.	
Write 7 as a fraction.	$\frac{7}{1}$
Write the reciprocal of $\frac{7}{1}$.	$\frac{1}{7}$
Check:	$7\cdot\left(\frac{1}{7}\right)$
	$1\checkmark$

Note:
Exercise:

Problem: Find the reciprocal:

ⓐ $\frac{5}{7}$
ⓑ $\frac{1}{8}$
ⓒ $\frac{11}{4}$
ⓓ 14

Solution:

ⓐ $\frac{7}{5}$
ⓑ 8
ⓒ $\frac{4}{11}$
ⓓ $\frac{1}{14}$

Note:
Exercise:

Problem: Find the reciprocal:

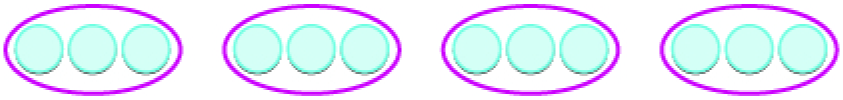
ⓐ $\frac{3}{7}$
ⓑ $\frac{1}{12}$
ⓒ $\frac{14}{9}$
ⓓ 21

Solution:

ⓐ $\frac{7}{3}$
ⓑ 12
ⓒ $\frac{9}{14}$
ⓓ $\frac{1}{21}$

Divide Fractions

Why is $12 \div 3 = 4$? We previously modeled this with counters. How many groups of 3 counters can be made from a group of 12 counters?



There are 4 groups of 3 counters. In other words, there are four 3s in 12.
So, $12 \div 3 = 4$.
For every division there is a related multiplication. In this case, $4 \cdot 3 = 12$.

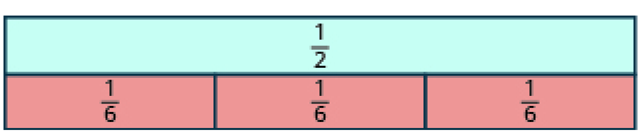
Division can be thought of as subtraction where the same number is subtracted over and over again. How many times can 3 be subtracted from 12?
$12 - 3 = 9. 9 - 3 = 6. 6 - 3 = 3. 3 - 3 = 0.$
This also gives the answer 4.

Dividing fractions is similar. Suppose we want to find the quotient: $\frac{1}{2} \div \frac{1}{6}$. We need to figure out how many $\frac{1}{6}$ there are in $\frac{1}{2}$. We can use fraction tiles to model this division. We start by lining up the half and sixth fraction tiles as shown in the next figure. Notice, there are three $\frac{1}{6}$ tiles in $\frac{1}{2}$, so $\frac{1}{2} \div \frac{1}{6} = 3$.The related multiplication problem is $3 \cdot \frac{1}{6} = \frac{1}{2}$.

Thought of as repeated subtraction, the first step is: $\frac{1}{2} - \frac{1}{6}$.
This requires a common denominator which is 6. The problem now becomes:
$\frac{3}{6} - \frac{1}{6} = \frac{2}{6}$.

$\frac{2}{6} - \frac{1}{6} = \frac{1}{6}$.

$\frac{1}{6} - \frac{1}{6} = 0$.
This also gives the answer 3.

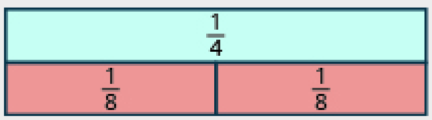


Example:
Exercise:

Problem: Model: $\frac{1}{4} \div \frac{1}{8}$. Show the corresponding multiplication.

Solution:
Solution

We want to determine how many $\frac{1}{8}$ are in $\frac{1}{4}$. Start with one $\frac{1}{4}$ tile. Line up $\frac{1}{8}$ tiles underneath the $\frac{1}{4}$ tile.



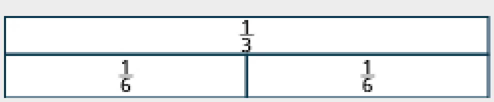
There are two $\frac{1}{8}$ in $\frac{1}{4}$.

So, $\frac{1}{4} \div \frac{1}{8} = 2$. The corresponding multiplication is $2 \cdot \frac{1}{8} = \frac{1}{4}$.

Note:
Exercise:

Problem: Model: $\frac{1}{3} \div \frac{1}{6}$. Show the corresponding multiplication.

Solution:

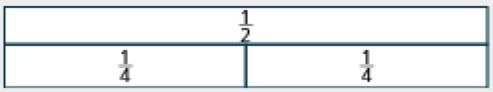


The corresponding multiplication is $2 \cdot \frac{1}{6} = \frac{1}{3}$.

Note:
Exercise:

Problem: Model: $\frac{1}{2} \div \frac{1}{4}$. Show the corresponding multiplication.

Solution:



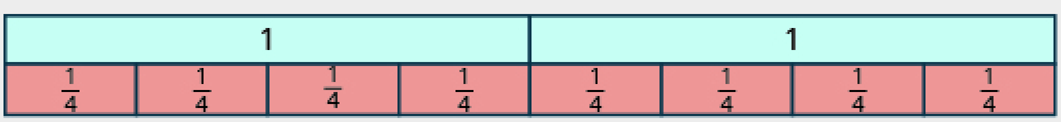
The corresponding multiplication is $2 \cdot \frac{1}{4} = \frac{1}{2}$.

Example:
Exercise:

Problem: Model: $2 \div \frac{1}{4}$. Show the corresponding multiplication.

Solution:
Solution

We are trying to determine how many $\frac{1}{4}$ there are in 2. We can model this as shown.

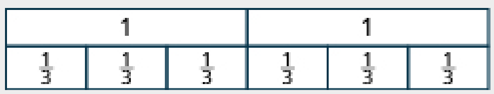


There are eight $\frac{1}{4}$ in 2.
$2 \div \frac{1}{4} = 8$.
The corresponding multiplication is $8 \cdot \frac{1}{4}$ =2.

Note:
Exercise:

Problem: Model: $2 \div \frac{1}{3}$ Show the corresponding multiplication.

Solution:

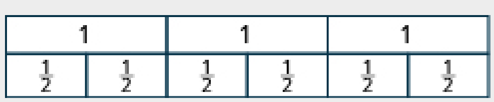


The corresponding multiplication is $6 \cdot \frac{1}{3} = 2$.

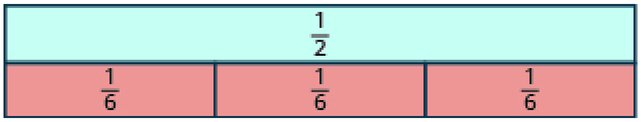
Note:
Exercise:

Problem: Model: $3 \div \frac{1}{2}$ Show the corresponding multiplication.

Solution:



The corresponding multiplication is $6 \cdot \frac{1}{2} = 3$.

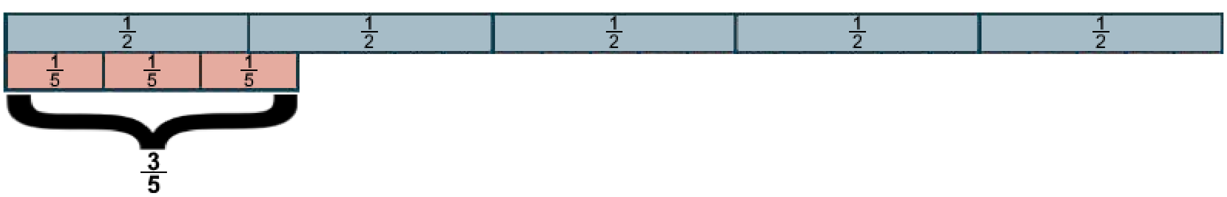


Recall $\frac{1}{2} \div \frac{1}{6}$.
Assume that we divided $\frac{1}{2}$ a foot by $\frac{1}{6}$ of a foot. The answer is still 3, not 3 feet but just the number 3. We can divide a $\frac{1}{2}$ foot long strip into 3 $\frac{1}{6}$ of a foot long strips. This is true of any other unit such as miles, pounds, or gallons. For example, if I have $\frac{1}{2}$ a pound of candy then I can divide it into 3 $\frac{1}{6}$ pound portions. The unit we use to measure doesn't matter as long as both parts are measured using the same unit.

Imagine if we measured in inches rather than in feet. 12 inches = 1 foot so $\frac{1}{2}$ foot = 6 inches and $\frac{1}{6}$ foot = 2 inches.
What is 6 inches divided by 2 inches? $6 \div 2 = 3$. This shows we can easily divide fractions if we can change the unit to get rid of the fractions. We are then left with a problem that just divides whole numbers.

The close relationship between division and subtraction means that we can use our knowledge of subtraction to help us with more difficult divisions. So far, our division of fraction problems have been easy. We've been able to illustrate them with tiles and they have worked out evenly. Let's try a harder problem: $\frac{5}{2} \div \frac{3}{5}$.

How much is left after we subtract: $\frac{5}{2} - \frac{3}{5}$?



In order to do the subtraction we need to find a common denominator. In this case, $2 \cdot 5 = 10$.

The next step is to change each fraction to an equivalent fraction with a denominator of 10.

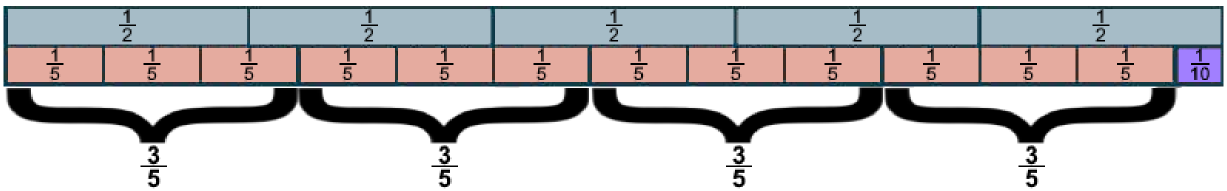
$\frac{5}{2} = \frac{5 \cdot 5}{2 \cdot 5} = \frac{25}{10}$ and $\frac{3}{5} = \frac{3 \cdot 2}{5 \cdot 2} = \frac{6}{10}$.

Subtracting $\frac{25}{10} - \frac{6}{10} = \frac{19}{10}$.

Again $\frac{19}{10} - \frac{6}{10} = \frac{13}{10}$.

Again $\frac{13}{10} - \frac{6}{10} = \frac{7}{10}$.

Again $\frac{7}{10} - \frac{6}{10} = \frac{1}{10}$ until we can no longer take away $\frac{6}{10}$.



We were able to subtract $\frac{3}{5}$ from $\frac{5}{2}$ 4 times and $\frac{1}{10}$ was left over. The 4 is easy to understand, but what about the part that remains? It is $\frac{1}{10}$ of a unit, but what we want to know is what part of $\frac{3}{5}$ it is.

Recall that $\frac{3}{5}$ is equivalent to $\frac{6}{10}$.

Thought of that way, the individual pieces are the same size, they are both $\frac{1}{10}$. We can easily tell that it takes 6 $\frac{1}{10}$ pieces to make $\frac{6}{10}$ so $\frac{1}{10}$ is $\frac{1}{6}$ of $\frac{6}{10}$. Therefore the final answer is 4 $\frac{1}{6}$.

Caution: 4 $\frac{1}{10}$ is not the answer. The $\frac{1}{10}$ is the size of the piece remaining, but not what part of $\frac{3}{5}$ that piece is.

If you are thinking that there must be an easier way, don't worry there is. Just like you wouldn't normally want to multiply fractions by repeated addition, you don't normally want to divide fractions by repeated subtraction. The important part is to notice that changing to a common denominator was an essential part of the solution.

Once we had changed to $\frac{25}{10}$ divided by $\frac{6}{10}$ we were almost done. Imagine that we had measured $\frac{25}{10}$ and $\frac{6}{10}$ in units that were $\frac{1}{10}$ the size. Then the amounts would have been 25 and 6 and we'd be dividing with whole numbers. Since the unit of measurement does not matter as long as it is the same for both parts, our answer would be $\frac{25}{6} = 4\frac{1}{6}$.

This agrees with the Equivalent Fraction Property.

If a, b, and c are numbers where $b \neq 0$ and $c \neq 0$, then

Equation:

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$$

Previously, we had thought of c as an integer, $c \neq 0$, but it can also be a fraction. Also remember that an equation can be used in either direction.

Equation:

Division of Fractions with a Common Denominator

$$\frac{a}{c} \div \frac{b}{c} = \frac{a}{b}$$

From the example above,

Equation:

$$\frac{25}{10} \div \frac{6}{10} = \frac{25}{6}$$

Dividing by 10 is the same as multiplying by $\frac{1}{10}$. In doing division of fractions, we are using the Equivalent Fraction Property going from the right side of the property to the left side. Once we have a common denominator, we can immediately write $\frac{25}{6}$.

An Intuitive Look at the Equivalent Fraction Property

How many 3 pound portions do I get when I divide 12 pounds of food?
$12 \div 3 = 4$.

What if I make both the portion size and total amount of food twice as big?
How many $3 \cdot 2 = 6$ pound portions do I get when I divide $12 \cdot 2 = 24$ pounds of food? $24 \div 6 = 4$.

What if I make both the portion size and total amount of food ten times as big?
How many $3 \cdot 10 = 30$ pound portions do I get when I divide $12 \cdot 10 = 120$ pounds of food? $120 \div 30 = 4$.

In each case the answer is 4 portions. Why? In each case the numerator and denominator were multiplied by the same factor. Therefore the Equivalent Fractions Property applies.

Make the portion size and the amount of food smaller by the same factor.

What if I make both the portion size and total amount of food half as big?
How many $3 \div 2 = \frac{3}{2}$ pound portions do I get when I divide $12 \div 2 = \frac{12}{2}$ pounds of food? $\frac{12}{2} \div \frac{3}{2} = 4$.

What if I make both the portion size and total amount of food ten times smaller?
How many $3 \div 10 = \frac{3}{10}$ pound portions do I get when I divide $12 \div 10 = \frac{12}{10}$ pounds of food? $\frac{12}{10} \div \frac{3}{10} = 4$.

In each case the answer is 4 portions. Why? In each case, the numerator and denominator were divided by the same factor. Dividing by a factor can be thought of as multiplying by 1 divided by the same factor.

Sometimes it is easier to see the math idea when we think of money.

How many $\$3$ candies can I buy if I have 15 one dollar bills? $\$15 \div \$3 = 5$

.
How many 75 cents = 3 quarters candies can I buy if I have 15 quarters? $\frac{15}{4}$ of a dollar $\div \frac{3}{4}$ of a dollar $= 5$.
How many 30 cents = 3 dime candies can I buy if I have 15 dimes? $\frac{15}{10}$ of a dollar $\div \frac{3}{10}$ of a dollar $= 5$.

Since the denominators are the same, the division of fractions was easy. We just divided the numerators.

Dividing Fractions with Different Denominators
Just as with adding and subtracting fractions with different denominators the first step is to get a common denominator.
It is not important if the denominator is the LCM, just as long as it is a common multiple. The fraction will normally be simplified at the end of the problem.

Example:
$\frac{2}{3} \div \frac{4}{5}$

A common denominator for 3 and 5 is 15. Change each fraction to equivalent fractions with a denominator of 15.

$\frac{2 \cdot 5}{3 \cdot 5} \div \frac{4 \cdot 3}{5 \cdot 3}$

By the commutative property $3 \cdot 5 = 5 \cdot 3$. We don't have to do the extra work of seeing that is 15 (although we already did.)
Therefore the fractions have the same denominator and the problem simplifies to $\frac{2 \cdot 5}{4 \cdot 3}$. This can be simplified to $\frac{5}{6}$.

Note:
Dividing Fractions Using the Reciprocal
If you previously learned how to divide fractions it is probably by multiplying by the reciprocal of the divisor.
If $a, b, c,$ and d are numbers where $b \neq 0, c \neq 0,$ and $d \neq 0$, then

Equation:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

We need to say $b \neq 0, c \neq 0$ and $d \neq 0$ to be sure we don't divide by zero.
To divide fractions, multiply the first fraction by the reciprocal of the second.

Why does this work?

Multiplying by the reciprocal is equivalent to finding a common denominator and simplifying.

Doing the multiplication: $\frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{c \cdot b}$, which is exactly the result from the common denominator method.

There is very little difference between the two methods and you can use the one you prefer now that you understand why they work.

Example:

Exercise:

Problem: Divide, and write the answer in simplified form: $\frac{2}{5} \div \frac{3}{7}$.

Solution:

Solution

	$\frac{2}{5} \div \frac{3}{7}$
Multiply the first fraction by the reciprocal of the second.	$\frac{2}{5} \cdot \frac{7}{3}$
Multiply.	$\frac{14}{15}$

Example:
Exercise:

Problem: Divide, and write the answer in simplified form: $\frac{2}{3} \div \frac{7}{5}$.

Solution:
Solution

	$\frac{2}{3} \div \frac{7}{5}$
Multiply the first fraction by the reciprocal of the second.	$\frac{2}{3} \cdot \frac{5}{7}$
Multiply.	$\frac{10}{21}$

Note:
Exercise:

Problem: Divide, and write the answer in simplified form: $\frac{3}{7} \div \frac{2}{3}$.

Solution:

$\frac{9}{14}$

Note:
Exercise:

Problem: Divide, and write the answer in simplified form: $\frac{2}{3} \div \frac{7}{9}$.

Solution:

$\frac{6}{7}$

Note:
Exercise:

Problem: Divide, and write the answer in simplified form: $\frac{3}{5} \div \frac{21}{10}$.

Solution:

$\frac{2}{7}$

Note:
Exercise:

Problem: Divide, and write the answer in simplified form: $\frac{7}{27} \div \frac{35}{36}$.

Solution:

$\frac{4}{15}$

Note:
Exercise:

Problem: Divide, and write the answer in simplified form: $\frac{5}{14} \div \frac{15}{28}$.

Solution:

$\frac{2}{3}$

Note:
ACCESS ADDITIONAL ONLINE RESOURCES

- Multiplying Fractions
- Dividing Fractions

Key Concepts

- **Equivalent Fractions Property**
  - If a, b, c are numbers where $b \neq 0$, $c \neq 0$, then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$ and $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$.
- **Simplify a fraction.**

  Rewrite the numerator and denominator to show the common factors.
  Simplify, using the equivalent fractions property, by removing common factors.
  Multiply any remaining factors.
- **Fraction Multiplication**

- If a, b, c, and d are numbers where $b \neq 0$and $d \neq 0$, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.

- **Reciprocal**
  - A number and its reciprocal have a product of 1.
- **Fraction Division**
  - If a, b, c, and d are numbers where $b \neq 0$, $c \neq 0$ and $d \neq 0$, then **Equation:**

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

  - To divide fractions, multiply the first fraction by the reciprocal of the second.

Exercises

Practice Makes Perfect

Multiply Fractions

In the following exercises, use a diagram to model.

Exercise:

Problem: $\frac{1}{2} \cdot \frac{2}{3}$

Solution:

$\frac{1}{3}$

Exercise:

Problem: $\frac{1}{2} \cdot \frac{5}{8}$

Exercise:

Problem: $\frac{1}{3} \cdot \frac{5}{6}$

Solution:

$\frac{5}{18}$

Exercise:

Problem: $\frac{1}{3} \cdot \frac{2}{5}$

In the following exercises, multiply, and write the answer in simplified form.

Exercise:

Problem: $\frac{2}{5} \cdot \frac{1}{3}$

Solution:

$\frac{2}{15}$

Exercise:

Problem: $\frac{1}{2} \cdot \frac{3}{8}$

Exercise:

Problem: $\frac{3}{4} \cdot \frac{9}{10}$

Solution:

$\frac{27}{40}$

Exercise:

Problem: $\frac{4}{5} \cdot \frac{2}{7}$

Exercise:

Problem: $\frac{2}{3}\left(\frac{3}{8}\right)$

Solution:

$\frac{1}{4}$

Exercise:

Problem: $\frac{3}{4}\left(\frac{4}{9}\right)$

Exercise:

Problem: $\frac{5}{9} \cdot \frac{3}{10}$

Solution:

$\frac{1}{6}$

Exercise:

Problem: $\frac{3}{8} \cdot \frac{4}{15}$

Exercise:

Problem: $\frac{7}{12}\left(\frac{8}{21}\right)$

Solution:

$\frac{2}{9}$

Exercise:

Problem: $\frac{5}{12}\left(\frac{8}{15}\right)$

Exercise:

Problem: $\left(\frac{14}{15}\right)\left(\frac{9}{20}\right)$

Solution:

$\frac{21}{50}$

Exercise:

Problem: $\left(\frac{9}{10}\right)\left(\frac{25}{33}\right)$

Exercise:

Problem: $\left(\frac{63}{84}\right)\left(\frac{44}{90}\right)$

Solution:

$\frac{11}{30}$

Exercise:

Problem: $\left(\frac{33}{60}\right)\left(\frac{40}{88}\right)$

Exercise:

Problem: $4 \cdot \frac{5}{11}$

Solution:

$\frac{20}{11}$

Exercise:

Problem: $5 \cdot \frac{8}{3}$

Exercise:

Problem: $\frac{3}{7} \cdot 21$

Solution:

9

Exercise:

Problem: $\frac{5}{6} \cdot 30$

Exercise:

Problem: $28 \left(\frac{1}{4} \right)$

Solution:

7

Exercise:

Problem: $51 \left(\frac{1}{3} \right)$

Exercise:

Problem: $8 \left(\frac{17}{4} \right)$

Solution:

34

Exercise:

Problem: $\frac{14}{5}(15)$

Exercise:

Problem: $1\left(\frac{3}{8}\right)$

Solution:

$\frac{3}{8}$

Exercise:

Problem: $(1)\left(\frac{6}{7}\right)$

Find Reciprocals

In the following exercises, find the reciprocal.

Exercise:

Problem: $\frac{3}{4}$

Solution:

$\frac{4}{3}$

Exercise:

Problem: $\frac{2}{3}$

Exercise:

Problem: $\frac{5}{17}$

Solution:

$\frac{17}{5}$

Exercise:

Problem: $\frac{6}{19}$

Exercise:

Problem: $\frac{11}{8}$

Solution:

$\frac{8}{11}$

Exercise:

Problem: 13

Exercise:

Problem: 19

Solution:

$\frac{1}{19}$

Exercise:

Problem: 1

Solution:

1

Divide Fractions

In the following exercises, model each fraction division.

Exercise:

Problem: $\frac{1}{2} \div \frac{1}{4}$

Exercise:

Problem: $\frac{1}{2} \div \frac{1}{8}$

Solution:

4

Exercise:

Problem: $2 \div \frac{1}{5}$

Exercise:

Problem: $3 \div \frac{1}{4}$

Solution:

12

In the following exercises, divide, and write the answer in simplified form.
Exercise:

Problem: $\frac{1}{2} \div \frac{1}{4}$

Exercise:

Problem: $\frac{1}{2} \div \frac{1}{8}$

Solution:

4

Exercise:

Problem: $\frac{3}{4} \div \frac{2}{3}$

Exercise:

Problem: $\frac{4}{5} \div \frac{3}{4}$

Solution:

$\frac{16}{15}$

Exercise:

Problem: $\frac{4}{5} \div \frac{4}{7}$

Exercise:

Problem: $\frac{3}{4} \div \frac{3}{5}$

Solution:

$\frac{5}{4}$

Exercise:

Problem: $\frac{7}{9} \div \left(\frac{7}{9}\right)$

Exercise:

Problem: $\frac{5}{6} \div \left(\frac{5}{6}\right)$

Solution:

1

Exercise:

Problem: $\frac{5}{18} \div \left(\frac{15}{24}\right)$

Exercise:

Problem: $\frac{7}{18} \div \left(\frac{14}{27}\right)$

Solution:

$\frac{3}{4}$

Exercise:

Problem: $\frac{7}{12} \div \frac{21}{8}$

Exercise:

Problem: $\frac{5}{12} \div \frac{15}{8}$

Solution:

$\frac{2}{9}$

Exercise:

Problem: $5 \div \frac{1}{2}$

Exercise:

Problem: $3 \div \frac{1}{4}$

Solution:

12

Exercise:

Problem: $\frac{3}{4} \div (12)$

Exercise:

Problem: $\frac{2}{5} \div (10)$

Solution:

$\frac{1}{25}$

Exercise:

Problem: $18 \div \left(\frac{9}{2}\right)$

Exercise:

Problem: $15 \div \left(\frac{5}{3}\right)$

Solution:

9

Exercise:

Problem: $\frac{1}{2} \div \left(\frac{3}{4}\right) \div \frac{7}{8}$

Exercise:

Problem: $\frac{11}{2} \div \frac{7}{8} \cdot \frac{2}{11}$

Solution:

$\frac{8}{7}$

Everyday Math

Exercise:

Problem:

Baking A recipe for chocolate chip cookies calls for $\frac{3}{4}$ cup brown sugar. Imelda wants to double the recipe.

ⓐ How much brown sugar will Imelda need? Show your calculation. Write your result as an improper fraction and as a mixed number.

ⓑ Measuring cups usually come in sets of $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, and 1 cup. Draw a diagram to show two different ways that Imelda could measure the brown sugar needed to double the recipe.

Exercise:

Problem:

Baking Nina is making 4 pans of fudge to serve after a music recital. For each pan, she needs $\frac{2}{3}$ cup of condensed milk.

ⓐ How much condensed milk will Nina need? Show your calculation. Write your result as an improper fraction and as a mixed number.
ⓑ Measuring cups usually come in sets of $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, and 1 cup. Draw a diagram to show two different ways that Nina could measure the condensed milk she needs.

Solution:

- ⓐ $4\frac{2}{3} = \frac{8}{3} = 2\frac{2}{3}$
- ⓑ Answers will vary.

Exercise:

Problem:

Portions Don purchased a bulk package of candy that weighs 5 pounds. He wants to sell the candy in little bags that hold $\frac{1}{4}$ pound. How many little bags of candy can he fill from the bulk package?

Exercise:

Problem:

Portions Kristen has $\frac{3}{4}$ yards of ribbon. She wants to cut it into equal parts to make hair ribbons for her daughter's 6 dolls. How long will each doll's hair ribbon be?

Solution:

$\frac{1}{8}\text{ yard}$

Writing Exercises

Exercise:

Problem: Explain how you find the reciprocal of a fraction.

Exercise:

Problem:

Rafael wanted to order half a medium pizza at a restaurant. The waiter told him that a medium pizza could be cut into 6 or 8 slices. Would he prefer 3 out of 6 slices or 4 out of 8 slices? Rafael replied that since he wasn't very hungry, he would prefer 3 out of 6 slices. Explain what is wrong with Rafael's reasoning.

Exercise:

Problem:

Give an example from everyday life that demonstrates how $\frac{1}{2} \cdot \frac{2}{3}$ is $\frac{1}{3}$.

Solution:

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
simplify fractions.			
multiply fractions.			
find reciprocals.			
divide fractions.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

Glossary

reciprocal
: The reciprocal of the fraction $\frac{a}{b}$ is $\frac{b}{a}$ where $a \neq 0$ and $b \neq 0$.